Patent Exhaustion and Licensing in the Supply Chain

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Abstract

This paper analyzes private and social incentives to levy an ad valorem licensing fee in a supply chain governed by the legal principle of patent exhaustion. With perfect competition at the upstream and downstream stage, the choice of the licensing segment is irrelevant for the patent holder and consumers. When exactly one segment of the value chain is monopolistic while the other one is competitive, the patent holder prefers licensing at the monopolistic stage, leading to an alignment of private and social incentives. With imperfect competition at both stages, excessive downstream licensing can arise. We demonstrate that charging licensing fees at both stages of the supply chain ("double-dipping") can be profitable for the patent holder and beneficial for consumers. We discuss the implications of this result for the application of the patent exhaustion principle.

JEL classification: D43, L41, L44.

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1 Introduction

This paper studies the privately and socially optimal level of patent licensing in multi-tiered vertical supply chains. The legal principle of "patent exhaustion", also known as the "first sale doctrine" in copyright law, limits the control that a patent holder can exert over a patented product once it has been sold.[1] Following the sale of a patented item, the patent holder may forfeit the right to restrict or control its subsequent use, resale, or further distribution. The underlying rationale behind patent exhaustion is to prevent the perpetual control of a specific product by the patentee, promoting competition and innovation by facilitating the unrestricted flow of goods in the market post-initial sale. This legal principle serves to deter the practice of "double-dipping," where a patent holder seeks repeated licensing fees for the same patented technology.[2] In Quanta Computer et al v LG Electronics (2008), the US Supreme Court recently re-affirmed the principle of patent exhaustion and extended its applicability to various facets of patented technologies, including method-based patents.[3]

In the context of supply chains, patent exhaustion signifies that once a patentee grants a license to an upstream firm, the patent holder relinquishes the right to enforce the patent against producers in subsequent layers of the value chain. In contrast, when engaging in licensing agreements with downstream firms in practical scenarios, patent holders often include contractual provisions granting licensees so-called "have made" rights. This allows the licensed end-product manufacturer to exercise the right to *have* components produced, which, if made by upstream third-party manufacturers, might infringe upon the patent holder's rights. This indirect insulation from patent infringement raises a myriad of questions. A fundamental query is what factors drive both

[1]This principle was first expressed in the US Supreme Court case Adams v. Burke, 84 U.S. (17 Wall.) 453 (1873). Massachusetts undertaker Burke purchased patented coffin lids from patentee and manufacturer Lockhart & Seelye. Burke then used the lids in an area outside of Boston where Adams, the plaintiff in this case, was the assignee of the patent. Adams sued Burke. The Supreme Court argued in favor of Adams stating:

> *That is to say the patentee or his assignee having in the act of sale received all the royalty or consideration which he claims for the use of his invention in that particular machine or instrument, it is open to the use of the purchaser without further restriction on account of the monopoly of the patentees.*

[2]The scope and application of patent exhaustion can vary, and legal interpretations may differ based on jurisdiction and specific circumstances.

[3]In this case, Intel obtained a license from LG Electronics for patents covering methods used in the manufacturing of microprocessors. Quanta Computers purchased microprocessors from Intel for computer assembly but did not independently license the relevant patents from LG Electronics. In response, LG Electronics filed a lawsuit against Quanta for patent infringement. The U.S. Supreme Court's decision in favor of Quanta re-affirmed the principle of patent exhaustion and extended its scope to method patents.

private and social incentives to license a particular technology at different levels of the value chain. This aspect of patent licensing has been at the core of various antitrust and patent litigation cases and is a contentious policy issue. For example, consider the high-profile case Daimler v Nokia (C-182/21) referred to the Court of Justice of the European Union (CJEU). Nokia holds standard essential patents on 3G/4G wireless communication technology. They offered car maker Daimler license agreements including "have made" rights for its suppliers such as Continental, Huawei, Bosch and others, but refused to license those upstream firms directly. Nokia pursued infringement actions against Daimler in German courts. Daimler challenged Nokia's licensing practices both in the infringement litigation and before the European Commission, arguing together with its suppliers that supplier-level licensing is standard practice in the automotive industry. Nokia, by contrast, maintained that efficiency favored downstream licensing. The parties settled in June 2021, and all pending litigation was withdrawn before the CJEU could rule on the appropriate stage of patent licensing in supply chains.

To examine the implications of the licensing stage choice, we develop a simple model of a supply chain with two-stage production involving a component (incorporating a patented product or method) and the final product. Patent exhaustion introduces two primary options for the patent holder: licensing the component at the upstream stage or licensing the end product at the downstream stage. These represent distinct approaches to intellectual property licensing within supply chains. With per-unit licensing royalties, the choice of the licensing stage becomes inconsequential, as the patent holder generates identical licensing revenue regardless of whether upstream or downstream producers are licensed. Instead, we consider ad valorem licensing fees which have been used in all of the above mentioned FRAND and antitrust cases. There is also empirical evidence supporting this approach, demonstrating that royalties based on revenues are prevalent. For instance, Bousquet et al. [1998] examined a sample of 278 contracts and found that 225 included royalties, but only nine of them were paid per unit sold. Similarly, Hegde [2014] finds that in all 505 sampled license contracts from the pharmaceutical and biomedical industry, the licenses specify a royalty, expressed as a percentage of annual gross sales revenues. With revenue-based licensing fees, licensing at the downstream level seems attractive due to the higher values associated with end products compared to component products. However, royalty payments depend jointly on the royalty base and the royalty rate.[4] We therefore analyze the choice of licensing stage in vertical

[4]When royalty rates are externally constrained, for example, by FRAND commitments, industry practice, or courts in standard-essential patent disputes, downstream licensing has an obvious appeal because it applies a given rate to a broader royalty base. One contribution of our analysis is to show that this intuition need not survive once royalty rates are chosen optimally.

supply chains where the royalty rate is fully *endogenized.*

Our analysis begins with a benchmark scenario where both upstream and downstream stages are characterized by perfect competition. In this scenario, we establish a strong neutrality result where the choice of the licensing stage is irrelevant, and the patent holder can implement the full integration outcome. Upstream licensing is able to extract a higher royalty rate. Downstream licensing benefits from a higher royalty base. With perfect competition, these two effects cancel each other out. We then consider scenarios where one stage is monopolistic while the other remains competitive. In such cases, the patent holder prefers to license at the stage where market power resides, aligning perfectly with the incentives of a social planner.

We extend the analysis to encompass cases of imperfect competition with market power at both stages, utilizing a competition parameter approach in a more general framework. This approach encompasses all possible market structures, including oligopolistic competition. In this instance, the choice of licensing stage becomes contingent on the allocation of production costs and market power across stages. Licensing tends to occur at the layer with relatively lower production cost and relatively less competition. Our findings also reveal a potential misalignment between private and social incentives. The patent holder excessively chooses downstream licensing in situations where the gain from extracting a higher share of industry profits outweighs the gain from a lower retail price and higher output. This misalignment is more likely to occur if production cost and the degree of competition are similar across the two layers of the supply chain.

Finally, our analysis introduces a novel perspective on the concept of "double-dipping," a scenario where a patent holder seeks compensation at multiple stages of the supply chain for a single patented product or process. This often involves pursuing royalties or licensing fees from both component manufacturers and device manufacturers selling to end-users. As discussed, courts and regulatory bodies may be wary of permitting double-dipping, as it may be viewed as an unfair or excessive monetization of a single invention. Our analysis uncovers a potential positive role for double-dipping. This arises as a mechanism to mitigate overall price distortions in supply chains stemming from the successive exercise of market power at each stage. We find that double-dipping can be profitable for the patent holder and lead to lower prices for consumers. The paper thus offers novel insights on the legal principle of patent exhaustion and sheds light on the potential benefits of a nuanced approach to licensing strategies in multi-tiered supply chains.

Despite the prevalence of litigation concerning the licensing stage and the associated policy discussions, economic research with formal modeling is relatively scant. As in our paper, Layne-Farrar et al. [2014] derive a royalty allocation neutrality result, demonstrating that how royalty rates are divided along the production chain has no substantial impact on social welfare in the case of per-unit royalty rates, assuming Nash bargaining between upstream and downstream firms. The assumption of Nash bargaining implies that there is no money left on the table, allowing firms to internalize any externalities and collectively behave as if they were integrated. In contrast, our analysis considers a scenario with *ad valorem* royalty rates to better capture the contentious issues in recent cases and the potential for double marginalization. This departure from per-unit royalty rates introduces complexities and contributes to a more comprehensive understanding of the economic implications associated with the choice of licensing stage.

Llobet and Padilla [2016] compare the total value of the product and the value of the component as the base for a royalty in licensing contracts. They show that these two royalty bases are equivalent to using ad valorem and per-unit royalties. However, they consider a much simpler setting in which the upstream firm is the patent holder who controls both the component price and royalty rate. We consider a more complex situation with an independent intellectual property (IP) holder who determines the stage in the production process where patent licensing takes place, and their equivalence results do not hold. Llobet and Neven [2022] develop a model of patent licensing in a vertical chain with investments. They show that licensors prefer to license their patents downstream, whereas upstream and downstream producers prefer upstream licensing. When the licensor chooses the stage of licensing, the static deadweight loss is lower than in the case where the producers choose. However, this comes at the cost of discouraging investment and innovation by upstream and downstream producers.

We can also make a connection between our paper and the public finance literature that investigates a Leviathan government that maximizes tax revenue. This is because the IP holder's licensing revenue maximization is formally equivalent to the government's tax revenue maximization problem. Gaudin and White [2014], for instance, compare unit and ad valorem tax regimes under the assumption that the government seeks to maximize tax revenue and derive conditions under which the ad valorem tax regime welfare-dominates the unit tax regime. However, their focus is on the comparison of two different tax instruments. They do not consider a vertical supply chain and the issue of the optimal level of taxation does not arise.[5] As in this paper, Peitz and Reisinger [2014] consider a vertical supply chain and study taxation. However, in line with the

[5] There is a large literature analyzing the relative merits of ad valorem vs. unit tax in terms of welfare comparison. See Suits and Musgrave [1953] for a monopoly setting and Delipalla and Keen [1992] and Anderson et al. [2001] for oligopolistic and imperfectly competitive market settings.

previous literature in public finance, their focus is on the comparison of specific and ad valorem taxes. They show that tax revenues should be raised only through ad valorem taxes.

In terms of methodology, both Johnson [2017] and our paper use a competition (or conduct) parameters approach to analyze vertical relationships under conditions of bilateral imperfect competition. However, Johnson's work focuses on the comparison between two common business practices in online markets: the agency and the wholesale model. We adapt the competition parameter approach to allow for licensing across different layers of the supply chain.

The rest of the paper is organized as follows. In Section 2, we set up a model of patent licensing in the supply chain. In Section 3, we consider the benchmark case of perfect competition at both stages of production and establish a strong neutrality result. Sections 4 and 5, respectively, consider upstream and downstream monopoly while the other segment is competitive. Section 6 considers a competition parameter approach that encompasses imperfect competition at both stages. Section 7 discusses industry practices and policy implications. The last section follows with concluding remarks. All remaining proofs are in Appendix A. Appendix B provides an analysis of the same framework with per-unit licensing rates and establishes neutrality results.

2 Model of Licensing in a Supply Chain

We consider a vertical supply chain with upstream and downstream firms. One unit of input from an upstream firm is needed to produce one unit of final output at the downstream stage. Downstream demand by end consumers is given by $Q(p)$, where p is the downstream market price with $Q'(p) < 0$. For notational convenience, we use two related measures of demand sensitivity,

$$\lambda(p) \equiv -\frac{Q(p)}{Q'(p)} = \frac{p}{\varepsilon(p)} > 0,$$

where $\varepsilon(p)$ is the price elasticity and $\lambda(p)$ is the inverse semi-price elasticity of demand. If demand is derived from a distribution function reflecting consumer heterogeneity, then the measure $\lambda(p)$ is the Mills' ratio of this distribution.[6] We assume throughout the

[6]Suppose consumers are differentiated with respect to their willingness-to-pay θ which is distributed according to $F(\theta)$ and density $f(\theta)$. Demand is then given by $Q(p) = 1 - F(p)$ and $\lambda(p) = (1 - F(p))/f(p)$.

paper that the price elasticity of demand is weakly increasing in the market price.[7] This implies that the elasticity of Mills' ratio is less than one, $p\lambda'(p)/\lambda(p) \leq 1$. We further assume that both λ and $\lambda(p)(2-\lambda'(p))$ have slope strictly less than one, to ensure the quasi-concavity of profit functions below.[8]

The set-up involves a patent holder (PH) who owns a technology essential for the production of the final good. We assume that the technology is implemented first at the upstream level. The upstream firms, with a marginal cost denoted c, supply the technology and charge a linear wholesale price w to the downstream firms. The downstream firms have a marginal cost d in addition to the wholesale price. Let $p^m(k)$ denote the monopoly price, which maximizes industry profits $(p-k)Q(p)$ with a constant marginal cost k and solves

$$(p-k)Q'(p) + Q(p) = 0 \quad \text{or} \quad p^m(k) - k = \lambda(p^m(k))\,.$$

Whenever convenient, we use the inverse function of the monopoly price,

$$k(p) \equiv p - \lambda(p),$$

so that $k(p)$ is the marginal cost for which p is the monopoly price. Thus, $\lambda(p^m(k))$ represents the mark-up (i.e., price minus marginal cost) of the monopolist. Furthermore, let $\Pi(p) \equiv (p-c-d)Q(p)$ denote industry profits at the total marginal cost of the supply chain.

To address the discussions regarding the appropriate stage for patent licensing, we consider two scenarios. First, following the current legal landscape shaped by the first sale doctrine, we assume that licensing can only occur at a single stage within the supply chain. If licensing takes place at the upstream stage, the principle of patent exhaustion provides protection to downstream producers who purchase the input supplied by the licensed upstream firm. In contrast, if licensing occurs at the downstream stage, the licensed downstream firm is able to secure "have-made rights" from the patent holder, which can shield its unlicensed suppliers from infringement. Second, in order to discuss the efficiency of the first-sale doctrine in this set-up, we allow the patent holder to charge license fees to both layers of the supply chain. We refer to this scenario as "double-dipping" licensing.

[7]Alfred Marshall referred to this property as the "law of the elasticity of demand" (Marshall, 1890, book III, chapter 4, pp 103-105).

[8]The condition $\lambda'(p) < 1$ is equivalent to $\eta_\varepsilon(p) \equiv \frac{p\varepsilon'(p)}{\varepsilon(p)} > 1 - \varepsilon(p)$. The corresponding elasticity expression for $\frac{d}{dp}[\lambda(p)(2-\lambda'(p))] < 1$ is considerably less transparent.

Throughout the main part of the paper, we focus on the implementation of ad valorem licensing fees $r_u \in [0,1]$ and $r_d \in [0,1]$ for upstream and downstream firms, respectively.[9] This fee is applied to the revenue of the firm obtaining the license. Hence, the royalty base price for upstream firms is the wholesale price while for downstream firms it is the final consumer price. The analysis looks at the patent holder's preference regarding the licensing stage. We investigate how the preference changes with vertical market structures and whether the patent holder's preference aligns with that of a social planner.

3 Perfect Competition Benchmark

As a benchmark consider an industry with perfectly competitive upstream and downstream segments in the supply chain. We investigate market outcomes with upstream and downstream licensing, respectively, and first establish a strong neutrality result that shows that patent holder and consumers are indifferent with respect to the choice of the licensing stage.

Suppose the patent holder sets an ad valorem license fee of r_d for all downstream firms. Let w_d and p_d denote the upstream wholesale and downstream retail price with downstream licensing, respectively. Perfectly competitive upstream firms set their price equal to their marginal cost, which yields $w_d = c$. For a given retail price p_d, the downstream sector makes profits of

$$[p_d(1-r_d) - w_d - d]Q(p_d) = (1-r_d)[p_d - \frac{c+d}{1-r_d}]Q(p_d)$$

where $(c+d)(1-r_d)^{-1}$ can be interpreted as the "effective marginal cost" of the downstream sector in the presence of downstream licensing. In a perfectly competitive downstream equilibrium, the retail price is equal to this marginal cost. Now consider the patent holder's choice of the ad valorem royalty rate. Even though the patent holder chooses r_d, it is more convenient to view the patent holder as indirectly choosing p_d with r_d equilibrating according to the downstream equilibrium. Let $r_d(p_d)$ be the royalty rate that implements a downstream price p_d, which is given by

$$r_d(p_d) = 1 - \frac{c+d}{p_d}.$$

The patent holder's license revenue ($\mathcal{L}$) as a function of the implemented retail price is

[9] In Appendix B we solve the general competition parameter model introduced in Section 6 for per-unit royalty rates and show that the choice of the licensing stage is inconsequential both for the patent holder and consumers.

then

$$\pounds_d(p_d) = r_d(p_d)p_dQ(p_d) = (p_d - c - d)Q(p_d),$$

which equals the integrated industry profits evaluated at the retail price. The optimal downstream price thus satisfies $p_d = p^m(c + d) = p^*$.

Similarly, suppose the patent holder sets an ad valorem license fee of r_u for all upstream firms. Let w_u and p_u denote the wholesale and the retail price with upstream licensing, respectively. With perfect competition and upstream licensing, downstream firms set their price equal to their marginal input cost, that is, $p_u = d + w_u$. For a given retail price, the wholesale price is $w_u(p_u) = p_u - d$ and the upstream sector makes profits of

$$[w_u(p_u)(1 - r_u) - c]Q(p_u) = (1 - r_u)[p_u - d - \frac{c}{1 - r_u}]Q(p_u).$$

In a perfectly competitive upstream equilibrium, the profit margin is competed away, and the price is equal to the marginal cost of $d + c(1 - r_u)^{-1}$. Conversely, the licensing rate $r_u(p_u)$ which implements a given p_u is

$$r_u(p_u) = 1 - \frac{c}{p_u - d}.$$

For a given retail price p, the royalty rate with upstream licensing is strictly higher than with downstream licensing $r_u(p) > r_d(p)$. To extract rents, the patent holder must induce a higher retail price, which requires making the chain behave as if marginal cost exceeded $c + d$. Under upstream licensing, the effective marginal cost of the chain is $c/(1 - r_u) + d$ as the royalty rate only raises upstream cost. Under downstream licensing, the rate raises the full cost $c + d$ of the chain by a factor $1/(1 - r_d)$. The same royalty rate therefore has a weaker effect on the retail price upstream than downstream. As a result, the patent holder can set a higher rate under upstream licensing to implement the same target price.

We can then write the patent holder's licensing revenues as

$$\pounds_u(p_u) = r_u(p_u)(p_u - d)Q(p_u) = (p_u - c - d)Q(p_u).$$

It follows immediately that the optimal retail price satisfies $p_u = p^m(c + d) = p^*$ and we get the same profits as with downstream licensing. Hence, the following strong neutrality result holds with perfect competition at both stages of the supply chain.

Proposition 1. *Consider perfectly competitive upstream and downstream industries. The patent holder is indifferent between upstream and downstream licensing. The licensing stage does not affect consumer surplus.*

With perfectly competitive upstream and downstream segments, the patent holder can set licensing terms to implement the retail monopoly price $p^m(c+d)$ and extract the entire monopoly profit of a vertically integrated firm owning production and intellectual property. This is true irrespective of whether licensing occurs upstream or downstream.

The licensing stage affects only how the monopoly profit is extracted through the two components of the per-unit revenue (the royalty rate multiplied by the base). For a given final retail price, upstream licensing allows the patent holder to impose a higher royalty rate,

$$\frac{r_u(p)}{r_d(p)} = \frac{p}{p-d}.$$

This is the *rate extraction effect* which favors upstream licensing. The second term is the royalty base. Upstream licensing applies the royalty to $w = p - d$ whereas downstream licensing uses the retail price p. Since the retail price always exceeds the wholesale price, this term naturally favors downstream licensing because the royalty base is larger when royalties are levied on the final product rather than on the intermediate input. This is the *royalty base effect* of downstream licensing. In the benchmark case of perfect competition at both stages, these two effects exactly offset each other. We have

$$\frac{r_u(p)}{r_d(p)}\frac{w}{p} = \frac{p}{p-d}\frac{p-d}{p} = 1$$

Hence, the patent holder is indifferent between upstream and downstream licensing. For the same reason, allowing the patent holder to "double dip" by charging ad valorem fees at both stages does not create additional licensing revenue for the patent holder.[10]

4 Upstream Monopoly

We now consider alternative market structures where one stage of the supply chain is monopolistic while the other remains perfectly competitive. We first analyze the market structure with upstream monopoly and perfect competition downstream.

Suppose that the patent holder licenses downstream firms with an ad valorem rate of r_d. For a given wholesale price w_d set by the upstream monopolist, the competitive downstream sector implements a retail price p_d that satisfies $w_d = (1 - r_d)p_d - d$. Let the upstream monopolist implicitly choose p_d to maximize profits,

$$\max_{p_d} \quad (w_d - c)Q(p_d) = (1 - r_d)[p_d - \frac{c+d}{1-r_d}]Q(p_d).$$

[10]We discuss double-dipping in more detail in Section 6.3 below.

Taking the derivative with respect to p_d yields the monopoly price with a marginal cost of $(c+d)(1-r_d)^{-1}$ or

$$k(p_d) = \frac{c+d}{1-r_d}. \tag{1}$$

Let $\bar{r}_d(p_d)$ be the licensing rate that satisfies condition (1), for a given p_d, with equality. Relative to perfect competition at both stages, the price in (1) is higher due to the monopolistic mark-up. Conversely, the royalty rate for a given retail price, $\bar{r}_d(p_d)$, is lower compared to the previous section. We can then write the patent holder's revenue as a function of p_d as

$$\begin{aligned} \pounds_d(p_d) = \bar{r}_d(p_d)p_dQ(p_d) &= \frac{k(p_d)-c-d}{k(p_d)}p_dQ(p_d) \\ &= \frac{k(p_d)-c-d}{k(p_d)}\frac{p_d}{p_d-c-d}\,\Pi(p_d) \equiv s_d(p_d)\,\Pi(p_d), \end{aligned} \tag{2}$$

where $s_d(p_d)$ is the patent holder's share in the total industry profit.

Consider upstream licensing instead. With a perfectly competitive downstream segment, the retail price is equal to the sum of wholesale price and downstream marginal cost. For a given p_u the wholesale price is $w_u(p_u) = p_u - d$. The upstream monopolist then solves

$$\max_{p_u} \quad [(1-r_u)w_u(p_u) - c]Q(p_u) = (1-r_u)[p_u - \frac{c}{1-r_u} - d]Q(p_u)$$

which yields

$$k(p_u) = \frac{c}{1-r_u} + d. \tag{3}$$

Compare to condition (1) under downstream licensing. At equal royalty rates $r_d = r_u = r$, downstream licensing leads to a higher retail price as the perceived marginal cost of the supply chain is higher. Let $\bar{r}_u(p_u)$ be the licensing rate that satisfies (3) with equality. It follows that $\bar{r}_u(p) > \bar{r}_d(p)$. The patent holder then chooses p_u to maximize its revenues

$$\begin{aligned} \pounds_u(p_u) = \bar{r}_u(p_u)w_u(p_u)Q(p_u) &= \frac{k(p_u)-c-d}{k(p_u)-d}(p_u-d)Q(p_u) \\ &= \frac{k(p_u)-c-d}{k(p_u)-d}\frac{p_u-d}{p_u-c-d}\,\Pi(p_u) \equiv s_u(p_u)\,\Pi(p_u). \end{aligned} \tag{4}$$

We solve the patent holder's problems with downstream and upstream licensing in the appendix and summarize the results as follows.

Proposition 2. *Consider an upstream monopoly and perfect competition downstream.*

The patent holder prefers to license at the upstream stage. The equilibrium retail price is strictly less with upstream licensing compared to downstream licensing ($p_u^* < p_d^*$). Private and social incentives regarding the licensing stage are aligned.

A monopolistic upstream sector breaks the neutrality result of the competitive benchmark. Upstream licensing is now optimal for the patent holder, and yields higher consumer surplus and higher total welfare. The intuition for these results relates to the change in the per-unit revenue of the patent holder (royalty rate times base). The royalty base comparison remains exactly the same as in the benchmark case:

$$\frac{w}{p} = \frac{p-d}{p},$$

because the downstream sector is still competitive, so the gap between retail and wholesale price is simply the downstream marginal cost, $p - w = d$. Thus, the royalty base advantage of downstream licensing is unchanged.

The difference arises from the rate extraction effect. In this case,

$$\frac{r_u(p)}{r_d(p)} = \frac{k(p)}{k(p)-d} > \frac{p}{p-d},$$

where $k(p) = p - \lambda(p)$. This shows that the rate extraction advantage of upstream licensing becomes stronger relative to the competitive benchmark. The reason is the presence of upstream monopoly power, which generates double marginalization. Since the upstream monopolist already imposes a markup, the patent holder must reduce royalty rates for a given final price relative to the competitive benchmark. The stronger the monopoly distortion, the larger this adjustment must be. Importantly, this reduction is relatively larger under downstream licensing because downstream licensing effectively "grosses up" both upstream and downstream costs, whereas upstream licensing only grosses up the upstream cost. Put differently, licensing the competitive downstream firms adds an additional markup on top of the monopolist's pricing decision, worsening double marginalization. As a result, upstream licensing is more effective at managing the double marginalization problem because it places the royalty burden directly on the monopolistic stage where pricing power already resides. Consequently,

$$\frac{s_u(p)}{s_d(p)} = \frac{k(p)}{k(p)-d}\frac{p-d}{p} > 1,$$

so upstream licensing allows the patent holder to capture a larger share of total industry profit.

The price ranking follows naturally. With monopoly power in the supply chain, the implemented retail price exceeds the standard monopoly price $p^m(c+d)$ because of double marginalization. To implement the integrated monopoly price would require a zero royalty rate, leaving no licensing revenue for the patent holder. Since the patent holder captures a larger share of industry profit under upstream licensing, deviations from the integrated monopoly outcome become more costly. This induces the patent holder to choose a lower retail price (albeit still above the standard monopoly price) under upstream licensing than under downstream licensing.

While we have focused on single-layer licensing, our analysis is without loss of generality. Even if the patent holder is allowed to license at both layers, the patent holder will not choose such an option. We show in the Appendix that the patent holder imposes an ad valorem royalty fee only on the monopoly sector even if "double-dipping" is allowed.

5 Downstream Monopoly

Now consider the diametrically opposed market structure where the downstream segment is monopolistic and the upstream segment is perfectly competitive.

First suppose there is downstream licensing. With a fully competitive upstream sector, the unit wholesale price is $w_d = c$ and the downstream monopolist maximizes profits of

$$[(1-r_d)p_d - c - d]Q(p_d) = (1-r_d)[p_d - \frac{c+d}{1-r_d}]Q(p_d)$$

which yields the price implicitly given by (1). This means that the licensing revenues as a function of the implemented downstream price are the same as in (2) in the previous section. Hence, the patent holder faces the same problem with downstream licensing independent of whether there is a monopolist in the upstream or downstream segment.

Remark. *Suppose a patent holder is licensing the downstream segment. When only one stage of the vertical supply chain is monopolistic (whereas the other stage is perfectly competitive), the final end-user prices are the same regardless of where the monopoly power resides.*

With downstream licensing the royalty base and the effective cost of the supply chain (and thus the implemented royalty rate) are the same regardless of where the single monopolist is located. Hence, any difference in market outcomes relative to Section 4 has to be driven by upstream licensing.

Suppose the patent holder uses upstream licensing. With a perfectly competitive upstream segment, the wholesale price follows from $(1-r_u)w_u = c$. The downstream monopolist maximizes its profits,

$$[p_u - \frac{c}{1-r_u} - d]Q(p_u),$$

which yields the same price as with an upstream monopolist in (3). The royalty rate as a function of the price is $\bar{r}_u(p_u)$ and the corresponding wholesale price is given by

$$w_u(p_u) = \frac{c}{1-\bar{r}_u(p_u)} = k(p_u) - d.$$

The patent holder then maximizes the following licensing revenues

$$\begin{aligned}\pounds_u(p_u) = \bar{r}_u(p_u)w_u(p_u)Q(p_u) &= \frac{k(p_u)-c-d}{k(p_u)-d}(k(p_u)-d)Q(p_u) \\ &= \frac{k(p_u)-c-d}{p_u-c-d}\Pi(p_u) \equiv s_u^D(p_u)\Pi(p_u), \end{aligned} \tag{5}$$

where the share superscript D denotes downstream monopoly to distinguish it from the case analyzed in section 4.

Proposition 3. *Consider a perfectly competitive upstream and a monopolistic downstream market. The patent holder prefers to license at the downstream stage. The equilibrium retail price with downstream licensing is strictly lower than under upstream licensing ($p_d^* < p_u^*$). Private and social incentives concerning the choice of the licensing stage are aligned.*

A monopolistic downstream sector also breaks the neutrality result of the competitive benchmark model. We observe the converse results compared to the scenario of an upstream monopoly in Section 4. Facing a competitive upstream and monopolistic downstream industry, the patent holder prefers to license downstream. Moreover, downstream licensing leads to lower prices for consumers. Hence, private and social incentives are again aligned.

The rate extraction effect is the same as in the upstream monopoly case,

$$\frac{r_u(p)}{r_d(p)} = \frac{k(p)}{k(p)-d}.$$

What changes is the royalty base effect. With downstream monopoly power, the downstream monopolist adds its own markup, so the wedge between retail and wholesale price exceeds the downstream marginal cost

$$p - w = p - k(p) + d > d.$$

Hence,

$$\frac{w}{p} = \frac{k(p) - d}{p} < \frac{p - d}{p}.$$

This makes the royalty base advantage of downstream licensing stronger than in both the benchmark and the upstream monopoly case. Intuitively, for a given retail price, the downstream royalty base remains the full final price p, whereas the upstream royalty base becomes smaller because it must reflect the downstream monopolist's additional markup. Therefore,

$$\frac{s_u^D(p)}{s_d(p)} = \frac{k(p)}{k(p) - d} \frac{k(p) - d}{p} = \frac{k(p)}{p} < 1.$$

Thus, downstream licensing becomes preferable for the patent holder: the royalty base effect favoring downstream licensing dominates the rate extraction effect favoring upstream licensing.

The reason can be understood by comparing how monopoly power affects the two components. In the expression for the rate extraction ratio, monopoly power affects royalty rates under both licensing regimes. Since the presence of a monopolistic stage already creates double marginalization, the patent holder must reduce royalty rates under both upstream and downstream licensing in order to sustain a given final price. Thus, market power affects both the numerator and the denominator of the rate ratio, and the effect is partially offset when comparing the two regimes. In other words, monopoly power lowers both $r_u(p)$ and $r_d(p)$, so its impact on the ratio $\frac{r_u(p)}{r_d(p)}$ is muted.

By contrast, the royalty base effect operates asymmetrically. With downstream monopoly power, the downstream royalty base remains the full retail price p, while only the upstream royalty base is reduced. The downstream monopolist widens the gap between wholesale and retail prices, shrinking the wholesale-price base on which upstream royalties are collected. Unlike the rate effect, this change affects only one side of the comparison: downstream licensing preserves the full royalty base, whereas upstream licensing suffers from a smaller base. As a result, the royalty base effect becomes stronger and dominates the muted rate extraction effect.

The same logic also explains the price ranking. Since the patent holder captures a larger share of total industry profit under downstream licensing, it internalizes more of the inefficiency created by double marginalization and therefore chooses a lower retail price. Licensing the monopolistic stage directly reduces the overall distortion in the vertical chain, leading to lower prices and higher welfare. Additionally, as in the previous section, we find that multi-layer licensing (or "double-dipping") is dominated by licensing only to the monopolistic sector.

Overall, our analysis so far has shown that the patent holder and consumers prefer licensing the monopolistic segment of the supply chain. Given the patent litigation cases discussed in the Introduction, it is interesting to verify whether these incentives align with the monopolist in the supply chain. Consider a downstream monopolist and let $\Pi_u^D(p)$ and $\Pi_d^D(p)$ denote the firm's profit under upstream and downstream licensing, respectively. The equilibrium profit difference of the downstream monopolist can then be decomposed as

$$\Pi_u^D(p_u^*) - \Pi_d^D(p_d^*) = \big[(p_u^* - k(p_u^*))Q(p_u^*) - (p_d^* - k(p_d^*))Q(p_d^*)\big] + \bar{r}_d(p_d^*)(p_d^* - k(p_d^*))Q(p_d^*).$$

The first term is the price effect of licensing on downstream revenues. The second term is the royalty burden under downstream licensing as the downstream monopolist pays a share $\bar{r}_d(p_d^*)$ of its monopoly rent. At equal retail prices, the monopolist always prefers upstream licensing. However, since $p^m(c+d) < p_d^* < p_u^*$, downstream licensing introduces less price distortion at the retail level. The overall sign of the profit difference is therefore a priori ambiguous and depends on the shape of demand. The following result holds.

Corollary 1. *If demand $Q(p)$ is linear, constant-elasticity or exponential, the downstream monopolist strictly prefers upstream licensing.*

Hence, for some of the more commonly used demand functions, the royalty burden dominates the price effect. The downstream monopolist may therefore have a private incentive to press for supplier-level licensing even when downstream licensing is less distortionary for the final price.[11] This is very much in line with Daimler's patent litigation in the dispute with Nokia discussed in the Introduction. Daimler's position was that, in the automotive industry, suppliers should obtain licenses and deliver components free of third-party rights, whereas Nokia insisted on end-product licensing and pursued infringement actions against Daimler.

Finally, note that our results in this and the previous section are distinct from an explanation that is based on the minimization of transaction costs. All else equal, transaction costs increase with the number of licensing contracts that must be executed. Transaction cost considerations therefore favor licensing at the stage with fewer licensees, that is, the monopolistic sector; in the next section, we extend the framework to allow for imperfect competition at both stages of production.

[11] A similar result holds for the case of an upstream monopolist who prefers the competitive, downstream segment to be licensed if demand is linear, constant-elasticity or exponential. We thank an anonymous referee for pointing out this implication of our results.

6 Bilateral Imperfect Competition

6.1 Analytical Framework

We extend the previous analysis to a situation of imperfect competition at both layers of the supply chain. The analysis of bilateral imperfect competition generalizes our previous results and encompasses all previous cases as special ones. To analyze this general set-up, we adopt the competition parameter approach—originally introduced by Bresnahan [1989], modified by Genesove and Mullin [1998], and subsequently employed by, among others, Weyl and Fabinger [2013] and Johnson [2017]. This approach does not rely on specifications of a particular model. Instead, it postulates that the inverse demand elasticity rule of the Lerner index is adjusted by a parameter $\theta \in [0, 1]$, which captures the degree of collective market power of firms in the market, that is,

$$p - k = \theta \frac{p}{\epsilon_d} = \theta \lambda(p),$$

where k is a constant marginal cost and ϵ_d is the price elasticity of demand. This modelling approach nests a wide range of forms of imperfect competition models as discussed in detail in Weyl and Fabinger [2013]. We apply this parameter approach to a vertical supply chain with imperfect competition and licensing at both stages. Let $\theta_u \in [0, 1]$ and $\theta_d \in [0, 1]$ be the market power parameters for the upstream and downstream industries, respectively. A value of $\theta_u = 0$ ($\theta_d = 0$) is equivalent to a situation where the upstream (downstream) industry is perfectly competitive. Higher values of these parameters correspond to market structures with less competition and more collective market power. At $\theta_u = 1$ ($\theta_d = 1$), the upstream (downstream) segment is a monopoly.

As a general framework, let us analyze price formation in the supply chain for given licensing fees (r_u, r_d), allowing for the possibility of "double dipping," whereby the patent holder collects royalties at both stages of the vertical chain. First consider the downstream segment and let $m_d(p) = (1 - r_d)p - w - d$ be the margin of this layer when all upstream (downstream) firms charge the same price w (p). The aggregate downstream profit is $\pi_d(p) = m_d(p)Q(p)$, with derivative

$$\frac{\partial \pi_d(p)}{\partial p} = m_d'(p)Q(p) + m_d(p)Q'(p).$$

Setting the derivative equal to zero yields the downstream sector's first-order condition.

This implicitly defines the price set by the downstream sector as

$$\frac{m_d(p)}{m_d'(p)} = p - \frac{w+d}{1-r_d} = \theta_d \lambda(p).$$

Solving for w yields the wholesale price implementing a retail price p,

$$\bar{w}(r_d, p) = (1-r_d)[p - \theta_d \lambda(p)] - d.$$

This is the royalty base for upstream licensing.

Turning to the upstream segment, its margin can be written as $m_u(p) = (1 - r_u)\bar{w}(r_d, p) - c$, and its profit is given by $\pi_u(p) = m_u(p)Q(p)$. Introducing the conduct parameter θ_u, we define the pricing behavior of the upstream industry by the condition

$$\frac{m_u(p)}{m_u'(p)} = \theta_u \lambda(p).$$

Substituting the expression for the wholesale price $\bar{w}(r_d, p)$ and rearranging yields the retail price as a function of the two licensing fees:

$$K(p) \equiv p - \lambda(p)\Theta(p) = \frac{c + d(1-r_u)}{(1-r_d)(1-r_u)}. \tag{6}$$

The right-hand side of (6) represents the effective marginal cost implied by production costs and the two ad valorem licensing fees. The term $K(p)$ on the left-hand side is the bilateral imperfect competition analogue of $k(p)$. It characterizes the marginal-cost level required to implement retail price p once market power at both stages of the supply chain is taken into account. The term $K(p)$ incorporates an aggregate index of supply chain market power given by

$$\Theta(p) = \theta_d + \theta_u - \lambda'(p)\theta_d\theta_u = \theta_d + \theta_u[1 - \theta_d\lambda'(p)].$$

This index summarizes how market power at the two layers combines to determine the aggregate wedge between the retail price and underlying production costs, with the interaction term $-\lambda'(p)\theta_d\theta_u$ capturing the effect of sequential pricing and pass-through between the two layers of the supply chain. The index $\Theta(p)$ is symmetric and increasing in both the upstream and downstream competition parameters. If both layers are competitive, then $\Theta(p) = 0$, implying $K(p) = p$. If market power is concentrated in a single monopolistic layer, that is, $(\theta_d, \theta_u) = (1, 0)$ or $(0, 1)$, then $\Theta(p) = 1$ and $K(p) = k(p)$. More generally, $K(p)$ is larger (smaller) than $k(p)$ when the aggregate competition index $\Theta(p)$ is less (greater) than one. [12]

6.2 Single-layer licensing

We first consider single-layer licensing, in which the patent holder is permitted to license either the upstream sector or the downstream sector, but not both, consistent with the principle of patent exhaustion. In Subsection 6.3, we relax this restriction and allow for "double dipping," whereby the patent holder can charge strictly positive licensing rates to both layers of the vertical chain. With downstream licensing ($r_u = 0$), solving condition (6) for r_d yields

$$\bar{r}_d(p) = 1 - \frac{c+d}{K(p)}.$$

With upstream licensing ($r_d = 0$), the royalty rate r_u required to implement a given price p is

$$\bar{r}_u(p) = 1 - \frac{c}{K(p) - d}.$$

Using the generalized royalty rates, downstream licensing generates licensing revenue

$$\pounds_d(p) = \bar{r}_d(p)pQ(p) = \frac{K(p) - c - d}{K(p)} \frac{p}{p - c - d}\Pi(p) \equiv s_d(p)\Pi(p),$$

whereas $\Pi(p) \equiv (p - c - d)Q(p)$ denotes total industry profit.

By contrast, upstream licensing generates

$$\pounds_u(p) = \bar{r}_u(p)\bar{w}(0,p)Q(p) = \frac{K(p) - c - d}{K(p) - d} \frac{p - \theta_d\lambda(p) - d}{p - c - d}\Pi(p) \equiv s_u(p)\Pi(p).$$

Thus, under both licensing regimes, the patent holder captures a fraction of total industry profit, with the magnitude of the profit share depending on the interaction between royalty rates, royalty bases, and the distribution of market power across the two layers of the supply chain.

As a first step, compare licensing revenues for a given retail price. The patent holder prefers upstream licensing at price p whenever

$$\frac{s_u(p)}{s_d(p)} = \frac{K(p)}{K(p) - d} \frac{p - \theta_d\lambda(p) - d}{p} \geq 1. \tag{7}$$

The first term captures the rate-extraction advantage of upstream licensing. For a given

[12] The difference to the one-sided monopoly benchmark is determined by the interaction term in $\Theta(p)$. Holding $\theta_d + \theta_u = 1$, we have $\Theta(p) = 1 - \lambda'(p)\theta_d\theta_u$. Hence $K(p) < k(p)$ under log-concave demand $\lambda'(p) < 0$, while $K(p) > k(p)$ under log-convex demand $\lambda'(p) > 0$.

retail price, upstream licensing can sustain a higher royalty rate because the downstream cost d is not grossed up by the upstream royalty. The second term captures the corresponding royalty-base disadvantage, since the upstream royalty is levied on the wholesale price rather than on the final retail price. Condition (7) nests the results of Sections 3 to 5 as special cases and illustrates how bilateral imperfect competition modifies the trade-off between royalty rates and royalty bases. If both layers are perfectly competitive ($\theta_d = \theta_u = 0$), then $K(p) = p$, and the rate and base effects exactly offset each other, as in Section 3. If the downstream layer is competitive ($\theta_d = 0$) while the upstream layer is monopolistic ($\theta_u = 1$), then $K(p) = k(p)$, and the upstream royalty base becomes $p - d$. In this case, the upstream rate extraction advantage dominates, as in Section 4. By contrast, if the upstream layer is competitive while $\theta_d = 1$, the upstream royalty base becomes $k(p) - d$, so the downstream base advantage dominates.

With market power at both layers, however, the revenue ranking depends on the implemented retail price. Rearranging (7) together with the relationship $K(p) \equiv p - \lambda(p)\Theta(p)$ in (6), we can verify that upstream licensing dominates at a given retail price if and only if

$$\frac{d}{K(p)} \geq \frac{\theta_d}{\Theta(p)}.$$

The left-hand side represents the downstream cost share in the effective supply chain cost and captures the strength of the upstream rate extraction advantage. The right-hand side represents the downstream share of aggregate market power and measures the markup included in the retail price base but excluded from the upstream royalty base. Higher implemented prices make downstream licensing relatively more attractive as they reduce the downstream cost share more than the downstream market power share.[13] Let $\hat{p}$ denote the retail price at which the condition holds with equality. Upstream licensing yields higher revenue for prices below $\hat{p}$, whereas downstream licensing yields higher revenue for prices above $\hat{p}$. The optimal licensing choice therefore depends on where the regime specific equilibrium prices p_u and p_d lie relative to this threshold.

Next, compare the retail prices induced by the two licensing regimes. At the optimal price p_i, where $i \in \{u, d\}$, the patent holder equates the marginal proportional gain from increasing its share of industry profit with the marginal proportional loss in total industry profit arising from the higher retail price induced by double marginalization:

$$\eta_i(p_i) \equiv \frac{s_i'(p_i)}{s_i(p_i)} = -\frac{\Pi'(p_i)}{\Pi(p_i)}. \tag{8}$$

[13]This requires the condition $1 - \lambda'(p)\Theta(p) > 0$. A sufficient condition is $\lambda''(p) = 0$, under which the aggregate competition index $\Theta(p)$ is constant. This case includes linear demand, exponential demand, and constant-elasticity demand.

Since extracting a positive royalty share requires a retail price above the integrated monopoly benchmark, $p_i > p^m(c+d)$, both licensing regimes generate double marginalization. The relative magnitude of the pricing distortion beyond the monopoly price is governed by the difference in share semi-elasticities. More specifically, upstream licensing leads to a less elevated retail price if the following condition holds for any $p > p^m(c+d)$.

$$\eta_u(p) - \eta_d(p) = -\frac{dK'(p)}{K(p)[K(p)-d]} + \frac{d + \theta_d[\lambda(p) - p\lambda'(p)]}{p[p - \theta_d\lambda(p) - d]} \leq 0.$$

The first term reflects the rate-extraction channel. It is negative because upstream licensing does not gross up the downstream production cost, thereby reducing the patent holder's marginal incentive to raise the retail price relative to downstream licensing. The second term reflects the royalty-base channel. It is positive because the upstream royalty is levied on the narrower wholesale-price base, but this disadvantage becomes less severe as the implemented retail price increases. More specifically, the upstream royalty base is the retail price minus downstream margin and cost. When the retail price is low, these deductions constitute a relatively large share of the final price, so the upstream royalty base is substantially smaller than the downstream royalty base. As the retail price rises, however, those deductions account for a smaller fraction of the final price, causing the upstream royalty base to become closer to the downstream royalty base in relative terms.[14] Hence, the rate-extraction channel favors lower prices under upstream licensing, whereas the royalty-base channel favors lower prices under downstream licensing.

Proposition 4. *There exist a unique $d^* \geq 0$ and a unique $c^*(d) \geq 0$ such that if $d \geq d^*$ and $c \leq c^*(d)$, then upstream licensing leads to lower prices. Otherwise, downstream licensing leads to lower prices.*

Proposition 4 shows that, when both layers possess market power, the price ranking depends on the allocation of production costs across the supply chain. A positive downstream cost is necessary for upstream licensing to have any rate-extraction advantage. If d is small, avoiding the gross-up of downstream cost has little value, and downstream licensing induces a lower price for all values of c. The upstream cost c affects the comparison only through its effect on the implemented price level. When c is small, equilibrium prices are relatively low, so the ratio $d/K(p)$ is relatively large and the rate extraction effect is correspondingly strong. In this case, upstream licensing can extract rents while adding less to the double marginalization distortion than downstream licens-

[14]The numerator of the royalty-base term captures this marginal effect: it is the difference between the downstream and upstream royalty bases, adjusted for the fact that this difference itself varies with the retail price p. The denominator contains the retail base and the upstream wholesale-price base.

ing. As c rises, however, the implemented prices also rise, causing $d/K(p)$ to fall and weakening the upstream rate extraction advantage. The broader royalty base associated with downstream licensing then becomes the more effective instrument for extracting licensing revenue, making downstream licensing the less distortionary regime.

We can now return to the patent holder's choice of licensing stage by comparing the optimal retail prices induced under upstream and downstream licensing.

Proposition 5. *There exist a unique $d^{**} > 0$ and a unique $c^{**}(d)$ such that, if $d \geq d^{**}$ and $c \leq c^{**}(d)$, the patent holder prefers licensing the upstream segment; otherwise, the patent holder prefers licensing the downstream segment.*

The patent holder chooses upstream licensing when the rate-extraction advantage at the optimal prices is sufficiently strong relative to the royalty-base advantage of downstream licensing. This requires a sufficiently large downstream cost d and a sufficiently small upstream cost c. The reason is that the equilibrium prices p_u and p_d both increase with the upstream cost c, whereas the revenue indifference threshold $\hat{p}$ derived from the fixed-price comparison remains unchanged. When c is low, both prices lie below $\hat{p}$, where upstream licensing extracts a larger share of industry profit because its rate-extraction advantage dominates. When c is high, both prices lie above $\hat{p}$, where the broader royalty base under downstream licensing becomes more valuable and downstream licensing dominates. For intermediate values of c, it holds that $p_u < \hat{p} < p_d$. In this region, upstream licensing preserves a larger total industry profit by inducing a lower retail price, whereas downstream licensing gives the patent holder a larger share of industry profit through the broader retail price base. At $c = c^{**}(d)$, the profit-preservation advantage of upstream licensing is exactly offset by the larger revenue share associated with downstream licensing. Beyond this point, the patent holder switches from upstream to downstream licensing, resulting in a discrete increase in the equilibrium retail price.

Proposition 6. *It holds that $d^{**} > d^{*}$ and $c^{**}(d) < c^{*}(d)$ for all $d > d^{**}$. This yields the following results: (i) If $c \leq c^{**}(d)$, the patent holder's choice of upstream licensing is socially efficient. (ii) If $c \geq c^{*}(d)$, the patent holder's choice of downstream licensing is socially efficient. (iii) Otherwise, the patent holder chooses downstream licensing even though upstream licensing is socially optimal.*

The patent holder's licensing choice is therefore socially efficient except for intermediate allocations of production costs. In the region $c^{**}(d) < c < c^{*}(d)$, upstream licensing induces a lower retail price than downstream licensing, $p_u < p_d$, yet the patent holder nevertheless prefers downstream licensing. Consumers prefer upstream licensing in this region because it mitigates the double marginalization distortion and therefore results in a lower retail price. The patent holder also values the lower price because it preserves a larger total industry profit from which licensing revenue can be extracted. However, the patent holder is concerned not only with preserving total industry profit, but also with maximizing the share of that profit that can be extracted as licensing revenue. In the intermediate region, the broader royalty base associated with downstream licensing raises the patent holder's extractable share of industry profit sufficiently to outweigh the larger industry profit preserved under upstream licensing through the lower retail price. As a result, the patent holder places excessive weight on the royalty-base advantage of downstream licensing, generating an excessive private incentive to license the downstream segment relative to the socially optimal outcome.

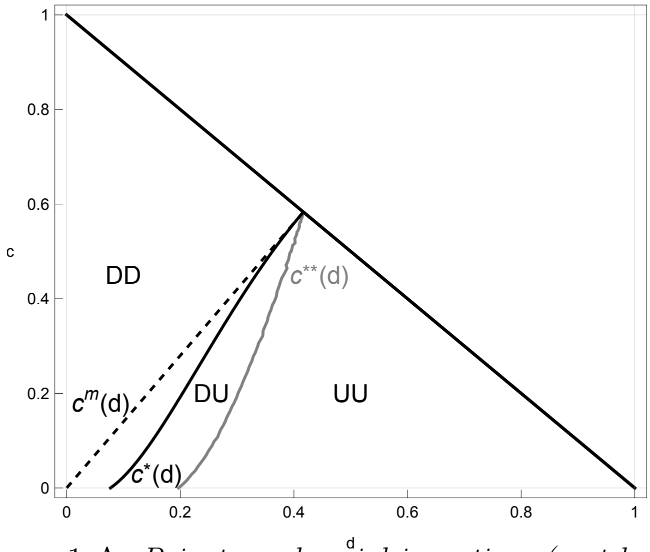

FIGURE 1.A: *Private and social incentives (cost levels).*

$[Q(p) = 1 - p,\ \theta_d = \theta_u = 0.4]$

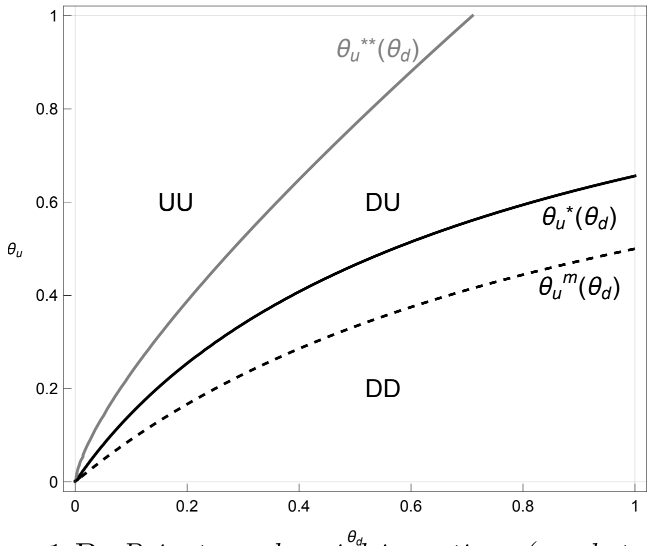

FIGURE 1.B: *Private and social incentives (market power)*

$[Q(p) = 1 - p,\ c = d = 0.2]$

FIGURE 1 illustrates the above results using a numerical example with linear demand. To account for the four main parameters of the model, Panel 1.A presents the results in (c, d) space, while Panel 1.B illustrates the results in terms of the market power parameters (θ_d, θ_u). Panel 1.A identifies the three regions characterized in Proposition 6. In *region DD*, the patent holder chooses downstream licensing, and downstream licensing also induces the lower retail price. In *region UU* the patent holder targets the upstream segment, which is also the socially efficient licensing stage. Inefficient downstream licensing arises in *region DU*, where upstream licensing would induce the lower retail price, but the patent holder nevertheless chooses downstream licensing. This misalignment occurs when the upstream and downstream sectors account for relatively similar shares of total production costs.

Panel 1.B provides the analogous comparison in market-power space. For given production costs, let $\theta_u^*(\theta_d)$ and $\theta_u^{**}(\theta_d)$ denote the values solving $c = c^*(d)$ and $c = c^{**}(d)$, respectively. Greater upstream market power strengthens the rate-extraction advantage of upstream licensing, whereas greater downstream market power strengthens the royalty-base advantage of downstream licensing. As a result, private and social incentives are aligned when market power is sufficiently concentrated at one layer of the supply chain. By contrast, misalignment is most likely when market power is more evenly balanced across the two layers. In that case, the patent holder prefers downstream licensing because it yields a larger extractable share of industry profit, even though upstream licensing would induce a lower retail price and therefore generate higher social welfare.

6.3 Double-dipping licenses

We now allow the patent holder to charge ad valorem royalties at both layers of the supply chain. Throughout this analysis, we impose $\lambda''(p) = 0$, so that the aggregate market-power index Θ is independent of the implemented retail price. This specification includes linear, exponential, and constant-elasticity demand.

For any upstream royalty rate r_u and implemented retail price p, the pricing condition (6) pins down the downstream royalty rate as

$$\hat{r}_d(r_u, p) = 1 - \frac{c + d(1 - r_u)}{(1 - r_u)K(p)}.$$

FIGURE 2 illustrates the corresponding iso-price locus, which nests both single-layer licensing regimes as special cases. At point A, there is exclusive downstream licensing with $\hat{r}_d(0, p) = \bar{r}_d(p)$. At point B, licensing occurs exclusively upstream with $r_d = 0$ and

$r_u = \bar{r}_u(p)$. Interior allocations along the iso-price locus correspond to "double dipping," where the patent holder collects royalties from both layers simultaneously. The iso-price locus is downward sloping and strictly concave: shifting royalty collection toward the upstream layer requires increasingly large reductions in the downstream royalty rate in order to maintain the same retail price.

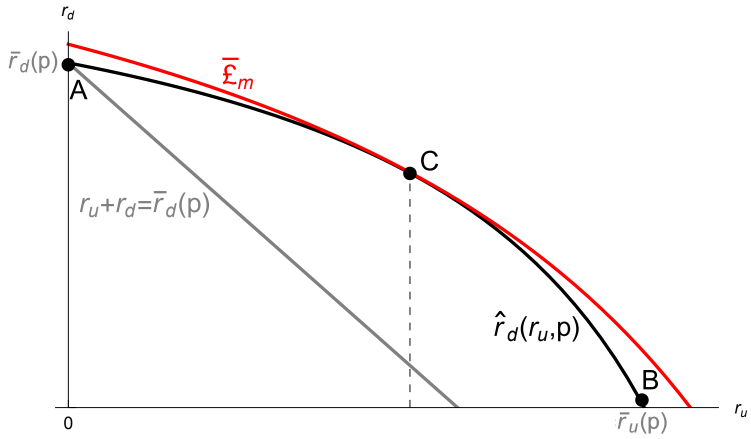

FIGURE 2: *Optimal Multi-Layer Licensing.*
$[Q(p) = 1 - p,\ c = d = 0.1,\ \theta_d = \theta_u = 0.4]$

Using $\hat{r}_d(r_u, p)$, the patent holder's revenue can be written as

$$£_m(r_u, p) = [r_u \bar{w}(\hat{r}_d(r_u, p), p) + \hat{r}_d(r_u, p)p]\ Q(p) = [s_d(p) + \Delta(r_u, p)]\ \Pi(p),$$

where

$$\Delta(r_u, p) = \frac{r_u \lambda(p)}{K(p)[p - c - d]} \left[d(\Theta(p) - \theta_d) - \frac{c\theta_d}{1 - r_u} \right].$$

The term $\Delta(r_u, p)$ measures the additional revenue share generated by introducing an upstream royalty while simultaneously reducing the downstream royalty so as to keep the retail price fixed. The bracketed expression isolates the central trade-off underlying double dipping. Introducing an upstream royalty creates an additional extraction channel, but it also requires the patent holder to reduce the downstream royalty rate and therefore give up some downstream licensing revenue. The gain from shifting royalty collection upstream is larger when downstream production costs d are more important and when upstream market power accounts for a larger share of the aggregate supply-chain distortion. By contrast, the cost of shifting royalties upstream becomes larger when upstream production costs c are high because these costs are grossed up by the upstream royalty, requiring a larger reduction in the downstream royalty rate to maintain the same retail price. Forgone downstream revenue is also more costly when downstream market power θ_d is high and the downstream royalty base more valuable.

For a given retail price, the patent holder chooses r_u to maximize $\Delta(r_u, p)$. The first-order condition equates the marginal gain from upstream rent extraction with the iso-price cost of reallocating royalties upstream:

$$d[\Theta(p) - \theta_d] = \frac{c\theta_d}{(1 - r_u)^2}. \tag{9}$$

Rearranging yields the optimal upstream royalty rate:

$$r_u^m(p) = 1 - \sqrt{\frac{c\theta_d}{d[\Theta(p) - \theta_d]}}.$$

A strictly positive upstream royalty is therefore optimal only when the rate extraction gain dominates the iso-price cost, that is,

$$c < c^m(d) \equiv \frac{d[\Theta - \theta_d]}{\theta_d}.$$

Point C in FIGURE 2 represents the tangency between the iso-price locus and an iso-revenue curve, where an interior double-dipping solution emerges. Substituting the optimal upstream royalty into the additional revenue share expression yields

$$\Delta_m(p) \equiv \Delta(r_u^m, p) = \frac{(r_u^m)^2 d\lambda(p)[\Theta - \theta_d]}{K(p)[p - c - d]} > 0,$$

whenever an interior solution exists. Hence, at any given retail price, double-dipping raises the patent holder's revenue share relative to exclusive downstream licensing.

Condition (9) also implies that exclusive upstream licensing is never optimal once double dipping is permitted. When upstream licensing is worth using $c < c^m(d)$, the patent holder optimally combines a positive upstream royalty with a positive downstream royalty rather than abandoning the broader downstream royalty base entirely. If $c \geq c^m(d)$, upstream licensing becomes too costly, and the patent holder chooses exclusive downstream licensing with $r_u = 0$.

As a final step, consider the optimal retail price under double-dipping:

$$\max_p [s_d(p) + \Delta_m(p)]\Pi(p).$$

Let $\eta_\Delta(p)$ denote the semi-elasticity of the additional revenue share with respect to price.[15] The first-order condition is then

$$\frac{s_d(p)}{s_d(p) + \Delta_m(p)}\eta_d(p) + \frac{\Delta_m(p)}{s_d(p) + \Delta_m(p)}\eta_\Delta(p) = -\frac{\Pi'(p)}{\Pi(p)}. \tag{10}$$

Thus, the marginal proportional gain from raising the retail price is a weighted average of the semi-elasticity of the downstream licensing share and the semi-elasticity of the additional revenue share generated by double dipping. Since $\eta_\Delta(p) < \eta_d(p)$, the overall share semi-elasticity under double dipping is lower than under exclusive downstream licensing. Double-dipping therefore weakens the marginal incentive to raise the retail price.[16] Intuitively, double dipping allows the patent holder to extract a larger share of industry profit while creating a smaller additional double marginalization distortion than exclusive downstream licensing.

Proposition 7. *Suppose $\lambda''(p) = 0$. If $c < c^m(d)$, the patent holder optimally charges positive royalty rates at both layers of the supply chain. If $c \geq c^m(d)$, the patent holder licenses only the downstream layer. Whenever the multi-layer solution is interior, allowing double-dipping lowers the retail price relative to exclusive downstream licensing.*

Proposition 7 explains why allowing double-dipping can benefit consumers. Under exclusive downstream licensing, the patent holder extracts revenue through the broad retail base, but this creates a strong incentive to raise the final price. Introducing an upstream royalty creates an additional extraction channel that allows the patent holder to obtain a larger revenue share at any given retail price. As a result, the marginal benefit from increasing the retail price is reduced. Double dipping therefore mitigates the double marginalization distortion relative to exclusive downstream licensing.

These results also have important implications for the application of the patent exhaustion principle in vertical supply chains. Suppose the principle is strictly enforced so that the patent holder is restricted to single-layer licensing and would therefore choose downstream licensing, as in region DD of Figure 1. This resembles Nokia's strategy in the Daimler dispute discussed above. Allowing royalties at both layers then lowers the final retail price whenever $c < c^m(d)$, or equivalently, in the market-power space of Panel 1.B, whenever $\theta_u > \theta_u^m(\theta_d)$. Hence, within this subset of the downstream-

[15]This means $\eta_\Delta(p) \equiv \Delta_m'(p)/\Delta_m(p)$.

[16]Since r_u^m is independent of p when $\lambda''(p) = 0$,

$$\eta_d(p) - \eta_\Delta(p) = \frac{K'(p)}{K(p) - c - d} + \frac{\lambda(p) - p\lambda'(p)}{p\lambda(p)} > 0,$$

because $K'(p) > 0$ and $p\lambda'(p)/\lambda(p) \leq 1$ by assumption.

licensing region, permitting double dipping increases consumer surplus by reducing the equilibrium retail price.

FIGURE 3 illustrates the pricing effects of multi-layer licensing using a linear-demand example. The grey lines represent the retail prices under exclusive upstream and downstream licensing, while the black line gives the equilibrium price under the single-layer restriction and the red line gives the equilibrium price under double-dipping. In the upper panel, with bilateral monopoly ($\theta_d = \theta_u = 1$), the patent holder switches from upstream to downstream licensing at $c^{**}(d)$, creating a discrete increase in the single-layer price. Double-dipping lowers the retail price whenever the patent holder would otherwise choose downstream licensing and the interior solution exists, that is, whenever $c < c^m(d)$. In the lower panel, where market power is weaker ($\theta_d = \theta_u = 0.4$), downstream licensing is privately optimal under the single-layer restriction for all values of c. In this case, allowing multi-layer licensing lowers the retail price throughout the interior region $c < c^m(d)$.

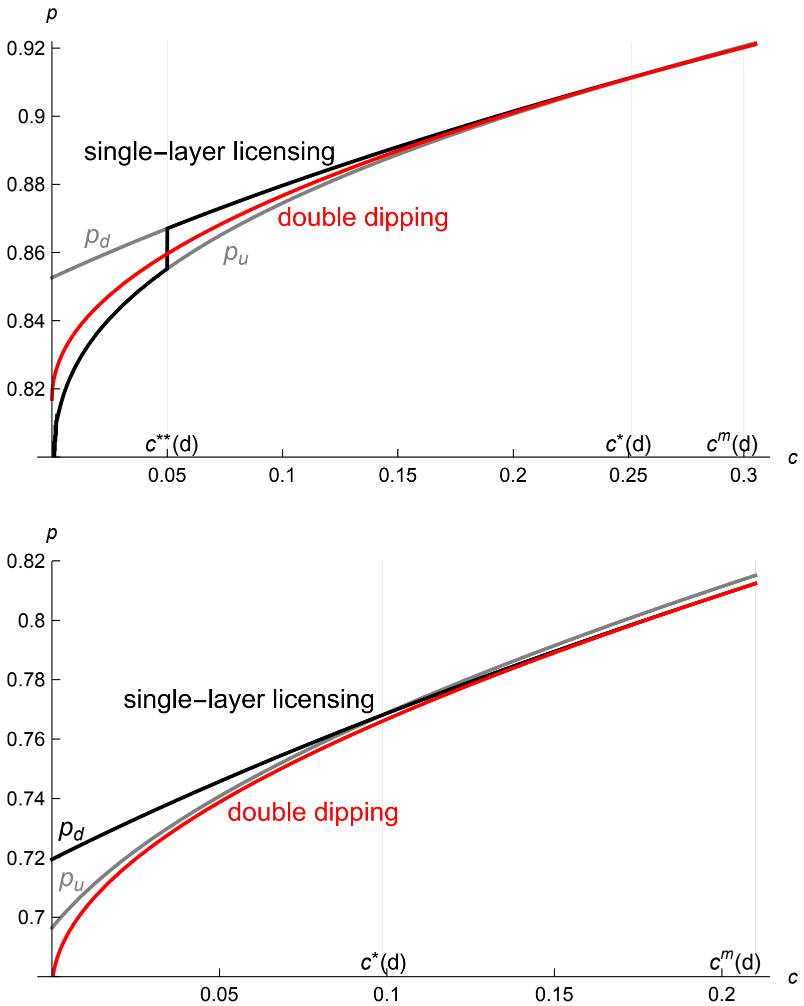

FIGURE 3: *Welfare Effect of Multi-Layer Licensing*

[$Q(p) = 1 - p$, $d = 0.15$, upper panel: $\theta_d = \theta_u = 1$, lower panel $\theta_d = \theta_u = 0.4$]

7 Industry Practices and Policy Implications

We have analyzed the optimal choice of licensing stage under the legal principle of patent exhaustion when the patent holder employs ad valorem licensing. In this section, we discuss how the insights from our model can be reconciled with observed industry practices by incorporating practical considerations that we have deliberately abstracted from in our analysis, but that may nevertheless play an important role in determining the appropriate licensing stage in real-world supply chains.

7.1 Per-Unit Licensing Fees

We have shown that the choice of licensing stage matters when market power is present at either level of the vertical supply chain. A natural question is whether the same mechanism operates under a per-unit royalty scheme. We find that it does not: under per-unit royalties, the choice of licensing stage is irrelevant. Both the retail price and the patent holder's profit are invariant to whether licensing occurs upstream or downstream (see Appendix B for details). Thus, concerns over the choice of licensing stage arise only under an ad valorem regime. This observation is consistent with the fact that most real-world disputes over the appropriate licensing stage arise in contexts where royalties are based on the value of the product.

7.2 Entire Market Value, Royalty Base, and Judicial Guidance

One common argument for downstream licensing is that end-product licensing applies royalties to a larger revenue base because final products are more valuable than intermediate components. For example, in testimony in *Ericsson Inc. v. D-Link Systems, Inc.* (E.D. Tex., 2013), Ericsson executive Christina Petersson stated that "one big advantage" of licensing end products is that "the royalty income will be higher since we calculate the royalty on a more expensive product."

However, from an economic perspective, focusing only on the royalty base is incomplete because royalty payments depend jointly on the royalty base and the royalty rate. This point has been emphasized by the Federal Circuit in *Lucent Technologies, Inc. v. Gateway, Inc.*, 580 F.3d 1301 (Fed. Cir. 2009), and *Ericsson, Inc. v. D-Link Systems, Inc.*, 773 F.3d 1201 (Fed. Cir. 2014). In Lucent, the court emphasized that there is "nothing inherently wrong" with using the value of the entire product as the royalty base so long as the royalty rate appropriately reflects the contribution of the patented component. Similarly, Ericsson v. D-Link stressed that the legally relevant issue is "the ultimate combination of royalty base and royalty rate." This judicial guidance strongly supports our modeling approach of fully endogenizing ad valorem royalty rates rather than treating the royalty rate as fixed.

7.3 FRAND and Cap on Revenue Share

Our analysis assumes that licensing fees are fully flexible. However, under Fair, Reasonable, and Non-Discriminatory (FRAND) licensing obligations — typically associated with standard essential patents (SEPs) — royalty terms may be constrained by courts or regulatory authorities. FRAND commitments require patent holders to offer licenses on terms that are not excessive or exclusionary and that do not discriminate across similarly situated licensees. In practice, this can translate into judicial scrutiny of royalty rates, benchmarks based on comparable licenses, or even implicit caps on the permissible royalty burden.

When the patent holder cannot charge the unconstrained optimal royalty due to court-imposed or institutionally determined limits, the effective royalty rate becomes partially exogenous. Under such constraints, the choice of licensing stage tends to favor downstream, end-product licensing, as the constrained rate is applied to a larger revenue base. As a result, FRAND-type restrictions can tilt incentives toward downstream licensing purely through a revenue-base effect — even in cases where upstream licensing would be preferred under fully flexible pricing, and a fortiori when the two are otherwise equivalent.

7.4 Downstream Licensing as Implicit Price Discrimination

A relevant feature of many vertical supply chains is that upstream inputs are relatively homogeneous, while downstream products are differentiated and span a wide range of qualities and prices. For example, in the handset industry, smartphones exhibit substantial vertical differentiation, with prices reflecting differences in quality, features, and brand positioning. In such an environment, downstream licensing with an ad valorem royalty can effectively serve as a substitute for explicit price discrimination across heterogeneous downstream firms. By tying royalties to the value of the final product, the patent holder can extract higher payments from high-end devices and lower payments from low-end devices without needing to design separate contracts for each segment. This provides a potential economic rationale for the preference for downstream, end-product licensing observed in practice. In particular, Qualcomm has historically licensed its standard essential patents at the handset level using percentage-of-device-price royalties, a practice that has been extensively discussed in both policy debates and litigation.[17]

7.5 Practical Considerations Beyond Pricing

Our model focuses on pricing distortions, but practical considerations may also shape the appropriate licensing stage. One important factor is technological expertise. Smartphone OEMs may be better positioned to understand and negotiate over cellular technologies because connectivity is central to mobile devices. In contrast, automobile manufacturers incorporating wireless connectivity may lack comparable expertise, making component-level licensing more attractive. This distinction is emphasized by Geradin and Katsifis [2021]. Transaction costs also matter. Licensing may be more efficient at the stage where fewer firms must be licensed, which often favors the monopolistic or more concentrated segment of the supply chain. These practical considerations may reinforce — or sometimes overturn — the purely pricing-based logic developed in our model.

8 Concluding Remarks

This paper develops a framework for analyzing the optimal licensing stage in vertical supply chains under the legal principle of patent exhaustion. We show that the choice of licensing stage depends critically on market structure and the form of royalty payments. When only one segment of the supply chain is monopolistic, the patent holder prefers to license that stage under ad valorem royalties, and this choice aligns with social efficiency. When both stages are imperfectly competitive, however, the patent holder may excessively favor downstream licensing, creating a divergence between private and social incentives. Our analysis also offers a new perspective on double dipping. While often viewed as unjustified repeated monetization of the same invention, multi-layer licensing can reduce double marginalization and improve welfare when both stages possess market power. In such cases, allowing limited double dipping may enhance both patent-holder incentives and consumer welfare.

Our analysis offers avenues for extension in several directions. First, we have assumed that patent holders do not participate directly in the production supply chain and have abstracted from vertically integrated firms. Extending our analysis to incorporate patent

[17] See, for example, FTC v. Qualcomm Inc. and Apple Inc. v. Qualcomm Inc., as well as analyses such as Geradin and Katsifis [2021], which document and discuss Qualcomm's licensing model and its implications.

holders with a presence in one of the production stages would be worthwhile. This extension could involve exploring the incentives for licensing in cases where patent holders are integrated into the production chain, as, for example, in the Qualcomm case (F.T.C. v. QUALCOMM INC. Case No. 17-CV-00220-LHK, 411 F. Supp. 3d 658, 2019), where Qualcomm faced accusations regarding its “No License, No Chips” policy. In instances where licensee firms are vertically integrated, licensing at the upstream stage may give rise to strategic transfer pricing for internal transactions, introducing additional layers of complexity. Second, our analysis has largely treated patents as ironclad and has not explicitly modeled incentives to litigate or invalidate patents. In reality, patents are inherently probabilistic, and litigation plays a central role in the enforcement of intellectual property rights, especially in industries characterized by frequent patent disputes and overlapping patent portfolios. Incorporating endogenous litigation decisions, patent validity challenges, and bargaining under legal uncertainty into a framework with general distributions of market power across vertical stages would provide a richer understanding of licensing incentives and remains an important avenue for future research.

Appendix A

Proof of Proposition 1

Consider double-dipping with $(r_u > 0, r_d > 0)$. With perfect competition, the upstream wholesale and downstream final prices are determined by

$$w = \frac{c}{1 - r_u} \quad \text{and} \quad p = \frac{w + d}{1 - r_d}$$

which yield the relationship between retail price and licensing rates,

$$r_d = 1 - \frac{c + d(1 - r_u)}{p(1 - r_u)}. \tag{11}$$

The patent holder’s profits as a function of the implemented downstream price p can be written as

$$\begin{aligned}
£_{dd}(p) &= [r_u w + r_d p]\, Q(p) \\
&= [r_u \frac{c}{1 - r_u} + (1 - \frac{c + d(1 - r_u)}{p(1 - r_u)})p]Q(p) \\
&= [\frac{r_u c p}{p(1 - r_u)} + \frac{(p - d)(1 - r_u) - c}{p(1 - r_u)}p]Q(p) = (p - c - d)Q(p)
\end{aligned}$$

Hence, the optimal retail price and overall licensing revenues are the same as with exclusive upstream or downstream licensing. There is a continuum of (r_u, r_d) combinations satisfying (11) for $p = p^m(c+d)$ that implement the optimal price. The upstream and downstream licensing fee imposed at only one stage are special cases. The neutrality result for perfectly competitive market thus extends to multi-layer licensing. QED.

Proof of Proposition 2

1. Price comparison. Use the licensing revenue definitions from the main text. Since p_u maximizes $\pounds_u(p)$,

$$\pounds'_u(p_u) = s'_u(p_u)\Pi(p_u) + s_u(p_u)\Pi'(p_u) = 0,$$

or

$$\Pi'(p_u) = -\frac{s'_u(p_u)}{s_u(p_u)}\Pi(p_u).$$

Substituting into $\pounds'_d(p_u)$ yields

$$\pounds'_d(p_u) = \Pi(p_u)s_d(p_u)\left[\frac{s'_d(p_u)}{s_d(p_u)} - \frac{s'_u(p_u)}{s_u(p_u)}\right].$$

A direct calculation gives

$$\frac{s'_d(p)}{s_d(p)} - \frac{s'_u(p)}{s_u(p)} = \frac{d\Big[(p-d)\big(pk'(p)-k(p)\big) + k(p)\big(p-k(p)\big)\Big]}{p(p-d)k(p)(k(p)-d)} > 0.$$

By $pk'(p) \geq k(p)$ and $p > k(p)$, the numerator is strictly positive. Hence $\pounds'_d(p_u) > 0$. From quasi-concavity of $\pounds_d$ follows that $p_u < p_d$. Moreover, since

$$\pounds'_u(p^m(c+d)) = \frac{k'(p^m(c+d))}{c}\big(p^m(c+d)-d\big)Q\big(p^m(c+d)\big) > 0,$$

it holds that

$$p^m(c+d) < p_u < p_d.$$

2. Patent-holder incentives. For any $p > p^m(c+d)$, we have $s_u(p) > s_d(p)$ and thus $\pounds_u(p) > \pounds_d(p)$, so the patent holder strictly prefers upstream licensing.
3. Non-optimality of double-dipping. Consider the patent holder engaging in double-dipping with $(r_u, r_d) > 0$. In the presence of monopoly power at the upstream level, the upstream wholesale and downstream final prices are determined by

$$p = \frac{w+d}{1-r_d}$$

Hence, the upstream monopolist's profits are

$$
\begin{aligned}
&[(1-r_u)w - c]\,Q(p) = [(1-r_u)((1-r_d)p - d) - c]Q(p) \\
&= (1-r_u)(1-r_d)[p - \frac{c}{(1-r_u)(1-r_d)} - \frac{d}{1-r_d}]Q(p)
\end{aligned}
$$

The first-order condition for profit maximization is then simply given by

$$
k(p) = \frac{c}{(1-r_u)(1-r_d)} + \frac{d}{1-r_d}. \tag{12}
$$

Licensing revenues are given by

$$
\pounds_{dd}(p) = [r_u w + r_d p]\,Q(p) = \{r_u[(1-r_d)p - d] + r_d p\}Q(p).
$$

Now consider an alternative licensing scheme that imposes a royalty rate only on the monopolistic upstream sector with $(r_u', r_d' = 0)$ that induces exactly the same final price p. Then, we have the following relationship.

$$
\frac{c}{1-r_u'} + d = \frac{c}{(1-r_u)(1-r_d)} + \frac{d}{1-r_d}
$$

or

$$
r_u' = 1 - \frac{c(1-r_u)(1-r_d)}{c + d(1-r_u)r_d}.
$$

With this alternative licensing scheme, the licensing income is

$$
\pounds_u(p) = r_u'(p-d)Q(p).
$$

Thus, upstream licensing weakly dominates if $\pounds_u(p) \geq \pounds_{dd}(p)$ or

$$
\begin{gathered}
\left[1 - \frac{c(1-r_u)(1-r_d)}{c + d(1-r_u)r_d}\right](p-d) - [r_u[(1-r_d)p - d] + r_d p] \geq 0 \quad \text{or} \\
\frac{dr_d(1-r_u)[(1-r_u)(p(1-r_d) - d) - c]}{c + d(1-r_u)r_d} = \frac{dr_d(1-r_u)[(1-r_u)w - c]}{c + d(1-r_u)r_d} \geq 0
\end{gathered}
$$

which holds for any $r_d \geq 0$. QED.

Proof of Proposition 3

1. Price comparison. The first-order condition for p_u yields

$$\frac{\partial \pounds_u}{\partial p_u} = (s_u^D)'(p_u)\Pi(p_u) + s_u^D(p_u)\Pi'(p_u) = 0,$$

or

$$\Pi'(p_u) = -\frac{(s_u^D)'(p_u)}{s_u^D(p_u)}\Pi(p_u).$$

Substituting into the derivative of $\pounds_d(p_u)$ yields

$$\frac{\partial \pounds_d}{\partial p_d}(p_u) = \Pi(p_u)s_d(p_u)\left[\frac{s_d'(p_u)}{s_d(p_u)} - \frac{(s_u^D)'(p_u)}{s_u^D(p_u)}\right].$$

A direct calculation gives

$$\frac{(s_u^D)'(p)}{s_u^D(p)} - \frac{s_d'(p)}{s_d(p)} = \frac{pk'(p) - k(p)}{pk(p)} > 0$$

as the numerator is assumed to be strictly positive. Hence,

$$\frac{s_d'(p_u)}{s_d(p_u)} - \frac{(s_u^D)'(p_u)}{s_u^D(p_u)} < 0,$$

and therefore $\pounds_d'(p_u) < 0$. From quasi-concavity of $\pounds_d$, it follows that $p_d < p_u$. Moreover, since

$$\frac{\partial \pounds_d}{\partial p_d}(p^m(c+d)) = \frac{k'(p^m(c+d))}{c+d}\, p^m(c+d)\, Q\big(p^m(c+d)\big) > 0,$$

it holds that

$$p^m(c+d) < p_d < p_u.$$

2. Patent-holder incentives. For any $p > p^m(c+d)$, we have $s_d(p) > s_u^D(p)$ and thus $\pounds_d(p) > \pounds_u(p)$, so the patent holder strictly prefers downstream licensing.

3. Non-optimality of double-dipping. Licensing revenues under double-dipping ($r_u > 0, r_d > 0$) are

$$\pounds_{dd}(p) = \left[\frac{r_u c}{1-r_u} + r_d p\right] Q(p).$$

Now consider an alternative licensing scheme that imposes a royalty rate only on the monopolistic downstream sector, $(r_u' = 0, r_d')$, and induces the same final price p. It must satisfy

$$\frac{c+d}{1-r_d'} = \frac{c}{(1-r_u)(1-r_d)} + \frac{d}{1-r_d},$$

or

$$r_d' = 1 - \frac{(c+d)(1-r_u)(1-r_d)}{c+d(1-r_u)}.$$

Hence,

$$\pounds_d(p) - \pounds_{dd}(p) = \left\{ \left[1 - \frac{(c+d)(1-r_u)(1-r_d)}{c+d(1-r_u)} \right] p - \left[\frac{r_u c}{1-r_u} + r_d p \right] \right\} Q(p).$$

Using

$$k(p) = \frac{c+d(1-r_u)}{(1-r_u)(1-r_d)},$$

this simplifies to

$$\pounds_d(p) - \pounds_{dd}(p) = \frac{c r_u}{1-r_u} \left(\frac{p}{k(p)} - 1 \right) Q(p) > 0.$$

Thus, exclusive downstream licensing weakly dominates double-dipping, with strict dominance whenever $r_u > 0$. QED.

Proof of Corollary 1

Linear demand. Let $Q(p) = a - bp$, so $\lambda(p) = (a-bp)/b$. Under upstream licensing, we get

$$p_u^* = \frac{3a + b(c+d)}{4b}, \qquad \Pi_D^u(p_u^*) = \lambda(p_u^*) Q(p_u^*) = \frac{(a - b(c+d))^2}{16b}.$$

Under downstream licensing, let $y \equiv a - bp_d^* \in (0, a/2)$. The patent holder's first-order condition is

$$b(c+d) = \frac{(a-2y)^3}{(a-y)^2 + y^2},$$

and the downstream monopolist's profit is

$$\Pi_D^d(p_d^*) = \frac{(c+d)y^2}{a-2y}.$$

Substituting out $c+d$ gives

$$\Pi_D^u(p_u^*) - \Pi_D^d(p_d^*) = -\frac{y^3\big(16y^3 - 24ay^2 + 15a^2y - 4a^3\big)}{4b((a-y)^2+y^2)^2}.$$

Now let

$$g(y) \equiv 16y^3 - 24ay^2 + 15a^2y - 4a^3.$$

Since

$$g'(y) = 48\Big(y - \frac{a}{2}\Big)^2 + 3a^2 > 0 \quad \text{and} \quad g\Big(\frac{a}{2}\Big) = -\frac{a^3}{2} < 0,$$

we have $g(y) < 0$ for all $y \in (0, a/2)$. Hence, $\Pi_D^u(p_u^*) > \Pi_D^d(p_d^*)$.

Constant-elasticity demand. Let $Q(p) = p^{-\varepsilon}$ with $\varepsilon > 1$. Then $\lambda(p) = p/\varepsilon$. Both licensing regimes yield the same first-order condition for the patent holder, so

$$p_u^* = p_d^* = \frac{\varepsilon^2}{(\varepsilon - 1)^2}(c + d).$$

Thus

$$\Pi_D^u(p_u^*) = \frac{{p_u^*}^{1-\varepsilon}}{\varepsilon}, \qquad \Pi_D^d(p_d^*) = \frac{(c+d)\, {p_d^*}^{-\varepsilon}}{\varepsilon - 1},$$

and therefore

$$\frac{\Pi_D^u(p_u^*)}{\Pi_D^d(p_d^*)} = \frac{p_u^*(\varepsilon - 1)}{\varepsilon(c + d)} = \frac{\varepsilon}{\varepsilon - 1} > 1.$$

Exponential demand. Let $Q(p) = e^{-\beta p}$. Then $\lambda(p) = 1/\beta$. Under upstream licensing,

$$p_u^* = c + d + \frac{2}{\beta}, \qquad \Pi_D^u(p_u^*) = \frac{1}{\beta} e^{-\beta(c+d)-2}.$$

Under downstream licensing, define $y \equiv \beta\Big(p_d^* - 1/\beta\Big) > 0$ giving first-order condition

$$\beta(c + d) = \frac{y^3}{y^2 + y + 1}, \quad \text{and} \quad \Pi_D^d(p_d^*) = \frac{e^{-1}}{\beta} \frac{y^2}{y^2 + y + 1} e^{-y}.$$

Hence

$$\log \frac{\Pi_D^u(p_u^*)}{\Pi_D^d(p_d^*)} = y - 1 - \frac{y^3}{y^2 + y + 1} + \log\Big(\frac{y^2 + y + 1}{y^2}\Big) \equiv h(y)$$

with $\lim_{y\to\infty} h(y) = 0$ and

$$h'(y) = -\frac{y^3 + y^2 + 2y + 2}{y(y^2 + y + 1)^2} < 0 \qquad \text{for all } y > 0,$$

Therefore $h(y) > 0$ for all $y > 0$, so $\Pi_D^u(p_u^*) > \Pi_D^d(p_d^*)$. QED.

Proof of Proposition 4

Let $\Lambda(p) \equiv \lambda(p)\Theta(p)$. The first-order condition with downstream licensing is

$$\begin{aligned}
\frac{\partial \pounds(0, \bar{r}_d, p)}{\partial p} = 0 &\iff \varepsilon_{r_d}(p) - \varepsilon_d(p) + 1 = 0 \\
&\iff \frac{(c+d)p[1-\Lambda'(p)]}{[p-\Lambda(p)][p-c-d-\Lambda(p)]} - \frac{p}{\lambda(p)} + 1 = 0 \\
&\iff p_d - c - d - \Lambda(p_d) = \lambda(p_d)\Phi_d(p_d), \qquad (13)
\end{aligned}$$

where

$$\Phi_d(p_d) \equiv \frac{p_d(c+d)[1-\Lambda'(p_d)]}{[p_d-\lambda(p_d)][p_d-\Lambda(p_d)]} > 0.$$

Similarly, the first-order condition with upstream licensing is

$$\begin{aligned}
\frac{\partial \pounds(\bar{r}_u, 0, p)}{\partial p} = 0 &\iff \varepsilon_{r_u}(p) - \varepsilon_d(p) + \varepsilon_w(p) = 0 \\
&\iff \frac{cp[1-\Lambda'(p)]}{[p-d-\Lambda(p)][p-c-d-\Lambda(p)]} - \frac{p}{\lambda(p)} + \frac{p[1-\theta_d\lambda'(p)]}{p-d-\theta_d\lambda(p)} = 0 \\
&\iff p_u - c - d - \Lambda(p_u) = \lambda(p_u)\Phi_u(p_u), \qquad (14)
\end{aligned}$$

where

$$\Phi_u(p_u) \equiv \frac{c[1-\Lambda'(p_u)][p_u-\theta_d\lambda(p_u)-d]}{[p_u-d-\Lambda(p_u)][p_u-\theta_d\lambda(p_u)-d-\lambda(p_u)(1-\theta_d\lambda'(p_u))]} > 0.$$

Let $p_d(c)$ and $p_u(c)$ be the solutions to (13) and (14), respectively. To ensure a revenue maximum, assume that both right-hand sides are decreasing in p whenever $p-c-d-\Lambda(p) > 0$. Inspection gives that $\partial\Phi_d(p)/\partial c > 0$ and $\partial\Phi_u(p)/\partial c > 0$ which implies that $\partial p_d/\partial c > 0$ and $\partial p_u/\partial c > 0$. Note that the lowest price $p_u(c=0)$ is given by

$$p - d - \lambda(p) - \lambda(p)\theta_d(1-\lambda'(p)) = 0.$$

We can thus focus below on the following condition

$$\theta_d \leq \frac{p-d-\lambda(p)}{\lambda(p)(1-\lambda'(p))} \equiv \bar{\theta}_d.$$

We first show that there can be at most one intersection of $p_d(c)$ and $p_u(c)$ as a function of c. It is sufficient to show that

$$\frac{\partial\Phi_d(p)}{\partial c} < \frac{\partial\Phi_u(p)}{\partial c}.$$

This is equivalent to

$$\frac{p}{[p-\lambda(p)][p-\Lambda(p)]}-\frac{p-\theta_d\lambda(p)-d}{[p-d-\Lambda(p)][p-d-\lambda(p)(\theta_d+1-\theta_d\lambda'(p))]}<0,$$

or

$$\begin{aligned}\Delta(\theta_d,\theta_u)\equiv p[p-d-\Lambda(p)][p-d-\lambda(p)(\theta_d+1-\theta_d\lambda'(p))]\\ -[p-\theta_d\lambda(p)-d][p-\lambda(p)][p-\Lambda(p)]<0.\end{aligned}$$

Taking the derivative of $\Delta(\theta_d,\theta_u)$ with respect to θ_u yields

$$\frac{\partial\Delta(\theta_d,\theta_u)}{\partial\theta_u}=\lambda(p)^2[1-\theta_d\lambda'(p)][d+\theta_d(\lambda(p)-p\lambda'(p))]>0,$$

since $\lambda(p)-p\lambda'(p)>0$. Hence Δ takes its maximum at $\theta_u=1$. Next,

$$\frac{\partial^2\Delta(\theta_d,1)}{(\partial\theta_d)^2}=2\lambda(p)^2[1-\lambda'(p)][\lambda(p)-p\lambda'(p)]>0,$$

so $\Delta(\theta_d,1)$ is convex in θ_d. Moreover,

$$\Delta(0,1)=-d[p(p-d)-\lambda(p)^2]<0,$$

since the term in brackets is strictly positive whenever $p-d>\lambda(p)$. Moreover, it holds that

$$\Delta(\bar{\theta}_d,1)=-\frac{d}{1-\lambda'(p)}[p-\lambda(p)][\lambda(p)-(p-d)\lambda'(p)]<0.$$

The bracket is strictly positive for $\lambda'(p)<0$. If $\lambda'(p)>0$, then

$$(p-d)\lambda'(p)<p\lambda'(p)<\lambda(p),$$

so the bracket is also strictly positive. It follows that $\partial\Phi_d(p)/\partial c<\partial\Phi_u(p)/\partial c$. This implies that $\partial p_d/\partial c<\partial p_u/\partial c$. Hence, if $p_d(0)<p_u(0)$, then there is no intersection and $p_d(c)<p_u(c)$ for all c. If instead $p_d(0)>p_u(0)$, then there exists exactly one intersection $c^*(d)>0$ such that if $c\leq c^*(d)$, then $p_u(c)\leq p_d(c)$; otherwise, if $c>c^*(d)$, then $p_u(c)>p_d(c)$.

It remains to characterize when this threshold exists. At $c=0$, upstream licensing induces a weakly lower price if and only if

$$[p_u(0) - \lambda(p_u(0))][p_u(0) - \Lambda(p_u(0))][p_u(0) - d - \Lambda(p_u(0))] \qquad (15)$$
$$\leq dp_u(0)\lambda(p_u(0))(1 - \Lambda'(p_u(0))).$$

Let Γ denote the right-hand side minus the left-hand side of (15). A direct calculation gives

$$\frac{\partial^2\Gamma}{(\partial d)^2} = 2[\Lambda(p) - \lambda(p)\Lambda'(p) - \theta_d\lambda(p)(1 - \lambda'(p))]$$
$$= 2\theta_u\lambda(p)\left[(1 - \lambda'(p))(1 - \theta_d\lambda'(p)) + \theta_d\lambda(p)\lambda''(p)\right] > 0$$

for any $\theta_u > 0$. From $1 - \Lambda'(p) > 0$ follows $(1 - \lambda'(p))^2 + \lambda(p)\lambda''(p) > 0$ which implies that the squared bracket is strictly positive. Thus Γ is strictly convex in d. Moreover,

$$\Gamma(0) = -\theta_d\lambda(p)[1 - \lambda'(p)][\lambda(p)(1 - \theta_u)(1 - \theta_d\lambda'(p))]^2 < 0$$

for any $\theta_d > 0$, while $\Gamma(d) \to \infty$ as d becomes large. Hence there exists a unique $d^* > 0$ such that if $d < d^*$, then $p_d(0) < p_u(0)$, and downstream licensing induces the lower price for all c. If $d \geq d^*$, then $p_u(0) \leq p_d(0)$, and the unique threshold $c^*(d)$ exists. This proves the proposition. QED.

Proof of Propositions 5 and 6

At a common implemented retail price p,

$$£_d(p) - £_u(p) = \frac{[p - c - d - \lambda(p)\Theta(p)][\theta_d(p - \lambda(p)\Theta(p)) - \Theta(p)d]}{[p - \lambda(p)\Theta(p)][p - d - \lambda(p)\Theta(p)]}Q(p)$$
$$= \frac{[K(p) - c - d][\theta_d K(p) - \Theta(p)d]}{K(p)[K(p) - d]}Q(p).$$

Thus the relevant roots are

$$K(p_1) = c + d, \qquad \theta_d K(\hat{p}) = \Theta(\hat{p})d.$$

On the relevant range with positive licensing revenues,

$$£_u(p) \geq £_d(p) \quad \Longleftrightarrow \quad p \in [p_1, \hat{p}].$$

The interval is non-empty iff

$$p_1 < \hat{p} \quad \Longleftrightarrow \quad c < \frac{\Theta(\hat{p}) - \theta_d}{\theta_d} d = \frac{\theta_u[1 - \theta_d \lambda'(\hat{p})]}{\theta_d} d.$$

From Proposition 4, $p_u(c)$ and $p_d(c)$ are increasing in c, whereas $\hat{p}$ is independent of c. If $\max\{p_u, p_d\} \leq \hat{p}$, then

$$\pounds_u(p_u) > \pounds_u(p_d) > \pounds_d(p_d),$$

so upstream licensing is optimal. If $\min\{p_u, p_d\} \geq \hat{p}$, then

$$\pounds_d(p_d) > \pounds_d(p_u) > \pounds_u(p_u),$$

so downstream licensing is optimal. It remains to consider $p_u < \hat{p} < p_d$. Let

$$D(c, d) \equiv \pounds_u(p_u) - \pounds_d(p_d).$$

By the envelope theorem, $\partial D / \partial c < 0$ whenever

$$Q(p_u) > Q(p_d) \frac{p_d[p_u - d - \Lambda(p_u)]}{[p_d - \Lambda(p_d)][p_u - d - \theta_d \lambda(p_u)]},$$

which holds because $p_u < p_d$, $Q(p_u) > Q(p_d)$, and the fraction is strictly less than one. Hence $D(c, d)$ is strictly decreasing in the intermediate region. The remaining ordering $p_d < \hat{p} < p_u$ cannot arise. If $p_d < p_u$, Proposition 4 implies that upstream licensing has the higher fixed-price revenue over the relevant interval below $\hat{p}$, so $\hat{p} > p_d$ implies $\hat{p} > p_u$. Therefore there exists a unique $c^{**}(d)$ such that

$$\pounds_u(p_u) \geq \pounds_d(p_d) \quad \Longleftrightarrow \quad c \leq c^{**}(d).$$

Moreover, this threshold can occur only when upstream licensing can induce the lower retail price, so

$$c^{**}(d) < c^*(d).$$

A positive $c^{**}(d)$ requires $p_u(0) \leq p_d(0)$. By Proposition 4 this holds iff $d \geq d^*$. Since $c^{**}(d) < c^*(d)$ and $c^*(d^*) = 0$, there exists $d^{**} > d^*$ such that

$$c^{**}(d^{**}) = 0.$$

Thus, if $d \geq d^{**}$ and $c \leq c^{**}(d)$, the patent holder chooses upstream licensing; otherwise it chooses downstream licensing. This proves Proposition 5. Proposition 6 follows from comparing the private threshold $c^{**}(d)$ with the price threshold $c^*(d)$. If $c \leq c^{**}(d)$, the patent holder chooses upstream licensing which induces the lower price. If $c \geq c^*(d)$, the patent holder chooses downstream licensing which induces the lower price. If

$$c^{**}(d) < c < c^*(d),$$

the patent holder chooses downstream licensing although upstream licensing induces the lower price. This is the excessive-downstream-licensing region. QED.

Proof of Proposition 7

Suppose $\lambda''(p) = 0$, so that Θ is independent of p. Using

$$\hat{r}_d(r_u, p) = 1 - \frac{c + d(1 - r_u)}{(1 - r_u)K(p)}$$

and

$$\bar{w}(\hat{r}_d(r_u, p), p) = \frac{c}{1 - r_u} + \frac{\lambda(p)[\Theta - \theta_d]}{K(p)} \left(\frac{c}{1 - r_u} + d\right)$$

the patent holder's revenue under multi-layer licensing is

$$\begin{aligned}
\pounds_m(r_u, p) &= [r_u \bar{w}(\hat{r}_d(r_u, p), p) + \hat{r}_d(r_u, p)p]\, Q(p) \\
&= \pounds_d(p) + [r_u \bar{w}(\hat{r}_d(r_u, p), p) + (\hat{r}_d(r_u, p) - \bar{r}_d(p))p]\, Q(p) \\
&= \pounds_d(p) + \left[r_u \left\{\frac{c}{1 - r_u} + \frac{\lambda(p)[\Theta - \theta_d]}{K(p)} \left(\frac{c}{1 - r_u} + d\right)\right\} - \frac{r_u c p}{(1 - r_u)K(p)}\right] Q(p) \\
&= \pounds_d(p) + \frac{r_u \lambda(p)}{K(p)} \left[d(\Theta - \theta_d) - \frac{c\theta_d}{1 - r_u}\right] Q(p) \\
&= \left[s_d(p) + \frac{r_u \lambda(p)}{K(p)[p - c - d]} \left\{d(\Theta - \theta_d) - \frac{c\theta_d}{1 - r_u}\right\}\right] \Pi(p) \\
&= [s_d(p) + \Delta(r_u, p)]\, \Pi(p).
\end{aligned}$$

For a fixed p, the choice of r_u maximizes $\Delta(r_u, p)$. The problem is concave since

$$\frac{\partial^2 \Delta(r_u, p)}{\partial r_u^2} = -\frac{2c\theta_d \lambda(p)}{K(p)[p - c - d](1 - r_u)^3} < 0.$$

Moreover, if

$$c \geq c^m(d) \equiv \frac{d(\Theta - \theta_d)}{\theta_d},$$

then $\partial\Delta/\partial r_u \leq 0$ at $r_u = 0$, and the patent holder sets $r_u = 0$. The results in the main text follow.

It remains to compare the double-dipping price with the exclusive downstream price. We have

$$\eta_d(p) = \frac{K'(p)}{K(p) - c - d} - \frac{K'(p)}{K(p)} + \frac{1}{p} - \frac{1}{p - c - d}$$

and

$$\eta_\Delta(p) = \frac{\lambda'(p)}{\lambda(p)} - \frac{K'(p)}{K(p)} - \frac{1}{p - c - d}.$$

Therefore,

$$\eta_d(p) - \eta_\Delta(p) = \frac{K'(p)}{K(p) - c - d} + \frac{\lambda(p) - p\lambda'(p)}{p\lambda(p)} > 0.$$

The first term is positive with positive licensing revenues; the second term is non-negative by $p\lambda'(p)/\lambda(p) \leq 1$.

At the exclusive downstream optimum p_d,

$$\eta_d(p_d) + \frac{\Pi'(p_d)}{\Pi(p_d)} = 0.$$

The log-derivative of the double-dipping objective at p_d is

$$\begin{aligned}
&\frac{d}{dp} \log\left([s_d(p) + \Delta_m(p)]\Pi(p)\right)\Big|_{p=p_d} \\
&= \frac{s_d(p_d)}{s_d(p_d) + \Delta_m(p_d)}\eta_d(p_d) + \frac{\Delta_m(p_d)}{s_d(p_d) + \Delta_m(p_d)}\eta_\Delta(p_d) + \frac{\Pi'(p_d)}{\Pi(p_d)} \\
&= \frac{\Delta_m(p_d)}{s_d(p_d) + \Delta_m(p_d)}\left[\eta_\Delta(p_d) - \eta_d(p_d)\right] < 0.
\end{aligned}$$

Let p_m be the optimal price under double-dipping. By quasi-concavity, the double-dipping optimum satisfies $p_m < p_d$. Finally, we show $c^m(d) \geq c^*(d)$. At $c = c^m(d)$,

$$c + d = \frac{d(\Theta - \theta_d)}{\theta_d} + d = \frac{d\Theta}{\theta_d}.$$

Since the same-price revenue-indifference price $\hat{p}$ satisfies $K(\hat{p}) = d\Theta/(\theta_d)$, and the no-licensing price p_1 satisfies $K(p_1) = c + d$, we have $p_1 = \hat{p}$ at $c = c^m(d)$. Both single-layer optimal prices are strictly above p_1. Hence, at $c = c^m(d)$, the relevant prices lie in the region where downstream licensing induces the lower retail price. By uniqueness of $c^*(d)$, $c^m(d) > c^*(d)$. Thus, whenever the multi-layer solution is interior, allowing double-dipping lowers the retail price relative to exclusive downstream licensing. QED.

Appendix B

Neutrality with Per-Unit Royalty Rates

Consider the competition parameter framework from Section 6, but suppose that the patent holder charges per-unit royalties f_u and f_d to the upstream and downstream sectors. Let

$$m_d(p) = p - w - d - f_d$$

be the downstream margin. The downstream pricing condition is

$$\frac{m_d(p)}{m_d'(p)} = p - w - d - f_d = \theta_d \lambda(p),$$

so

$$\bar{w}(p) = p - \theta_d \lambda(p) - d - f_d.$$

The upstream margin is

$$m_u(p) = \bar{w}(p) - c - f_u,$$

and the upstream pricing condition is

$$\frac{m_u(p)}{m_u'(p)} = \frac{p - \theta_d \lambda(p) - d - f_d - c - f_u}{1 - \theta_d \lambda'(p)} = \theta_u \lambda(p).$$

Rearranging gives

$$p - \lambda(p)\Theta(p) = c + d + f_u + f_d,$$

where

$$\Theta(p) = \theta_d + \theta_u - \lambda'(p)\theta_d\theta_u.$$

Thus the implemented retail price depends only on the total per-unit royalty

$$f = f_u + f_d,$$

not on how this royalty is allocated across the two layers. The patent holder's revenue is

$$\pounds(f_u, f_d) = (f_u + f_d)Q(p(f)) = fQ(p(f)).$$

Hence, for any total per-unit royalty f, all allocations (f_u, f_d) with $f_u + f_d = f$ generate the same retail price, the same licensing revenue, and the same consumer surplus. Optimizing over f therefore leaves the patent holder and consumers indifferent over the licensing stage. QED.

References

Simon P. Anderson, Andre de Palma, and Brent Kreider. Tax incidence in differentiated product oligopoly. *Journal of Public Economics*, 81(2):173–192, 2001.

Alain Bousquet, Helmuth Cremer, Marc Ivaldi, and Michel Wolkowicz. Risk sharing in licensing. *International Journal of Industrial Organization*, 16:535–554, 1998.

Timothy F. Bresnahan. Empirical studies of industries with market power. In Richard Schmalensee and Robert Willig, editors, *Handbook of Industrial Organization*, volume 2, pages 1011–1057. North-Holland, Amsterdam, 1989.

Sofia Delipalla and Michael Keen. The comparison between ad valorem and specific taxation under imperfect competition. *Journal of Public Economics*, 49(3):351–367, 1992.

Germain Gaudin and Alexander White. Unit vs. ad valorem taxes under revenue maximization. Available at SSRN, November 2014. URL `https://ssrn.com/abstract=2440708`. SSRN working paper, November 26, 2014.

David Genesove and Wallace P. Mullin. Testing static oligopoly models: Conduct and cost in the sugar industry, 1890–1914. *RAND Journal of Economics*, pages 355–377, 1998.

Damien Geradin and Dimitrios Katsifis. End-product- vs component-level licensing of standard essential patents in the internet of things context. Available at SSRN, May 2021. URL `https://ssrn.com/abstract=3848532`. SSRN working paper, May 18, 2021.

Deepak Hegde. Tacit knowledge and the structure of license contracts: Evidence from the biomedical industry. *Journal of Economics & Management Strategy*, 23(3), 2014. Fall 2014.

Justin P. Johnson. The agency model and MFN clauses. *Review of Economic Studies*, 84(3):1151–1185, 2017.

Anne Layne-Farrar, Gerard Llobet, and Jorge Padilla. Patent licensing in vertically disaggregated industries: The royalty allocation neutrality principle. *Communications and Strategies*, 95(3):61–84, 2014.

Gerard Llobet and Damien Neven. Investment and patent licensing in the value chain. Discussion Paper 17405, CEPR, 2022.

Gerard Llobet and Jorge Padilla. The optimal scope of the royalty base in patent licensing. *Journal of Law and Economics*, pages 45–73, 2016.

Martin Peitz and Markus Reisinger. Indirect taxation in vertical oligopoly. *Journal of Industrial Economics*, pages 709–755, December 2014.

Daniel B. Suits and Richard A. Musgrave. Ad valorem and unit taxes compared. *Quarterly Journal of Economics*, 67(4):598–604, 1953.

E. Glen Weyl and Michal Fabinger. Pass-through as an economic tool: Principles of incidence under imperfect competition. *Journal of Political Economy*, 121(3):528–583, June 2013.