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Abstract

We examine tippy network markets that accommodate price discrimination. The analysis shows that when a mild equilibrium refinement, the monotonicity criterion, is adopted, network competition may have a unique subgame-perfect equilibrium regarding the winner's identity; the prevailing brand may be fully determined by its product features. We bring out the concept of the dominant value margin, which is a metric of the effectiveness of divide-and-conquer strategies. The supplier with the larger dominant value margin may always sell to all customers in equilibrium. Such a market outcome is not always socially efficient since a socially inferior supplier may prevail if has a stand-alone-benefit advantage and only a modest network-benefit disadvantage.

JEL-Codes: L130, L400, D430.

Keywords: network externalities, equilibrium uniqueness, price discrimination, monotonicity criterion, dominant value margin, divide and conquer.

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1. INTRODUCTION

Many markets in the high-technology sector, as well as in other sectors, are characterized by network externalities; the value of a brand to a specific customer depends on the total number of customers that actually purchase the brand. Software, hardware, Internet products and telecommunications are some good examples. Then, as is well-known, the interdependence of customers may lead to the existence of multiple equilibria in which the role of expectations is crucial. The prevalence of a brand in the market may stem from a self-fulfilling prophecy, rather than from the brand's identity or product features; customers may buy a brand because they expect that other customers will also do so (e.g., Katz and Shapiro (1985), Farrell and Saloner (1986), Farrell and Klemperer (2007)). However, the multiplicity of equilibria and the marked randomness of the market outcome, — i.e., the ability of several competing brands to prevail in different equilibria regardless of their specific product features, — often make difficult the drawing of solid conclusions about business strategy or antitrust intervention.

We show that supplier competition in the presence of network externalities may have a unique subgame-perfect equilibrium regarding the winner's identity, and the prevailing supplier may be fully determined by the product features of its brand. Our analysis examines tippy network markets that are susceptible to winner-takes-all outcomes. In practice, several important network markets are tippy and end up being dominated by a single brand. As, for example, Shapiro and Varian (1999), among others, point out, network markets may tend to be tippy when customers do not have highly distinct needs and preferences, i.e., when there is sufficient preference homogeneity.

We focus on network markets that leave room for price discrimination. In our model suppliers are able to make discriminatory price offers (should they deem suitable), i.e., to sell to each customer at a different price.[1] Such a modeling feature is relevant empirically; in practice, several markets, — including (without being limited to) several network markets, — entail prices that are set through individual agreements or contracts among suppliers and customers, allowing suppliers to practice price discrimination if they

[1] The ability of suppliers to offer individual prices even when customers are homogeneous (thus extracting different rents from different customers) is a standard feature in the contract literature (e.g., Rasmusen, Ramseyer and Wiley (1991), Segal and Whinston (2000), Segal (2003)). Such ability to price discriminate is an important strategic tool that may assist suppliers to attain favorable or avoid unfavorable outcomes.

choose to do so. As a study of antitrust cases and hearings reveals (e.g., Baseman, Warren-Boulton and Woroch (1995), EC (2018)), price discrimination is often especially pronounced in network markets in the high-technology sector. Furthermore, high-technology network products are often marketed and distributed online. The special features of the Internet facilitate the provision of personalized, rather than generic, product packaging and the implementation of personalized pricing (Shapiro and Varian (1999)). In addition, a line of research suggests that the rise of artificial intelligence technology may possibly further facilitate the overall practice of price discrimination in the future (Shiller (2016), Milgrom and Tadelis (2019)).

We also adopt a mild refinement of subgame-perfection, called the monotonicity criterion. This condition is akin to the monotonicity property that is often used in social choice and political science to characterize desirable voting or social choice mechanisms (Moulin (1983), Arrow, Sen and Suzumura (2002)).[2] A monotonicity condition is also used in some articles in the literature on networks (e.g., Caillaud and Jullien (2001, 2003), Armstrong and Wright (2007)). In our model the refinement of the monotonicity criterion eliminates subgame-perfect equilibria in which agents rationally expect that an unanswered (by competitors) weak improvement, i.e., a weak reduction, in a supplier's price offers to all its equilibrium customers would lead to a smaller measure of such customers buying from the supplier. Such a criterion aims to eliminate unreasonable equilibria in which customers change their product choices for no apparent reason.

In the analysis suppliers have the opportunity to counter each other's price offers with divide-and-conquer strategies, making more lucrative price offers to some customers than to others. A supplier may be able to prevent expected subgames in which some customers buy from the competitor (enjoying related network externalities) by poaching some of the competitor's customers through substantially lower price offers than the competitor. The supplier may also simultaneously poach additional customers through progressively increasing price offers since each poached customer automatically increases the supplier's expected relative reservation value. When such a poaching strategy is profitable, a supplier may rule out equilibrium outcomes in which customers

[2] A social choice or voting mechanism is monotonic if additional support *ceteris paribus* for an otherwise winning candidate never turns winners into non-winners (Moulin (1983), Arrow, Sen and Suzumura (2002)). Monotonicity is a desirable characteristic of a mechanism.

buy from the competitor. Then, a supplier is less vulnerable to poaching by a competitor when its stand-alone benefit constitutes a larger fraction (and the network benefit a smaller fraction) of the reservation value in case it sells to the entire market; as the competitor poaches some of the supplier's customers, the remaining customers do not become much easier to poach since the supplier's reservation value does not decrease by much. Similarly, the supplier's own divide-and-conquer poaching strategy in case the competitor sells to some customers is facilitated since its large stand-alone benefit fully applies to all poached customers.

In line with such reasoning, we introduce the concept of the dominant value margin, i.e., the difference between the social value (or the social surplus) of a sale to a customer by the fully dominant brand and the fully dominated brand in a winner-takes-all outcome. The dominant value margin is shown to be a metric of the effectiveness of divide-and-conquer strategies; the profitability of a winning supplier depends on dominant value margins. Furthermore, it is shown that stand-alone benefits have a disproportionately large impact on dominant value margins relative to network benefits. The former are generated by both a fully dominant and a fully dominated brand (thus having a double impact on the difference between dominant value margins), while the latter stem only from a fully dominant brand that actually sells to customers. Then, regarding the winner's identity, there is a unique subgame-perfect equilibrium that meets the monotonicity criterion (although there may be multiple equilibrium prices by such a winning supplier); enjoying an advantage in divide-and-conquer poaching, the supplier with the larger dominant value margin always sells to all customers.

Since a comparison of dominant value margins, rather than of social welfare, determines the prevailing supplier, the market outcome is not necessarily socially optimal. A supplier may sometimes sell to all customers even if its competitor is socially superior in that the competitor generates higher social value per customer if it prevails. Such a socially inefficient outcome may occur when the socially inferior supplier enjoys a larger stand-alone benefit and also has a modest (but not too large) disadvantage in network benefits compared with its competitor. Then, given the disproportionate impact of stand-alone benefits on dominant value margins, the socially inferior supplier takes full advantage of its higher stand-alone benefit and prevails in equilibrium.

In practice, our analysis implies that in tippy network markets business strategies aiming to boost a brand's stand-alone benefit may be especially important for a brand's success (given the disproportionate impact of stand-alone benefits on dominant value margins). For example, as is well-known, one of the first commercial battles in a tippy network took place in the VCR market, where there were initially two different and incompatible formats, i.e., Beta (created by Sony) and VHS (created by JVC). Beta was introduced first and, as a first mover, it enjoyed early network externalities and stronger brand name recognition, which induced some market participants to believe that it would prevail. However, Beta was subsequently defeated by VHS, which completely dominated the market. The empirical analysis of Obashi (2003), which examines the VCR market in the U.S. from 1978 to 1986, finds that in the crucial period of 1978-1981, VHS enjoyed substantial stand-alone benefits in the eyes of consumers, which was a key factor in the prevalence of VHS. In particular, VHS tapes had a recording capacity of two hours, allowing consumers to record entire movies, while Beta tapes had a recording capacity of only one hour (Liebowitz and Margolis (1994)). The inability to record full movies was a significant shortcoming, rendering the stand-alone value of Beta tapes inferior in the eyes of consumers (despite other technical advantages, such as image and sound quality, that they offered). The outcome of the rivalry in VCRs was effectively sealed by 1982 (Obashi (2003)).[3]

Various business strategies that emphasize stand-alone benefits in network markets are often promoted by consultants and experts. Such strategies may be appealing even in the absence of network externalities. Our model, however, places special emphasis on them, displaying their key role in determining winners in tippy network markets. For example, a supplier may reward its customers in the network market by offering them the opportunity to buy outside goods at low prices (Shapiro and Varian (1999)); such a stand-alone perk is independent of the size of the network. Or, a supplier may attempt to raise rivals' stand-alone costs (since the stand-alone benefit is the net

[3] Furthermore, JVC promoted VHS through individualized business-to-business agreements or contracts at a wholesale level (which gave it ample opportunity to practice price discrimination should it deem appropriate). For example, JVC reached individual licensing agreements with several equipment manufacturers (such as Hitachi, Mitsubishi and Sharp) that agreed to adopt the VHS format. JVC also reached individual agreements with content providers (such as Magnetic Video Corporation of America) on the utilization of VHS in the provision of content (Cusumano, Mylonadis and Rosenbloom (1992)).

difference between stand-alone gains and stand-alone costs). A supplier, for instance, may strive to make itself easy to find by customers and its rivals difficult to find (Shapiro and Varian (1999)). In this regard Microsoft's strategy to ensure the pre-installment of Internet Explorer on PCs (Whinston (2001)), or Google's strategy to ensure the pre-installment of certain Google applications on smartphones (European Commission (2015)), may be interpreted as a means for increasing its stand-alone benefit relative to that of its rivals in that customers have to make special arrangements to sample or install rival products.[4]

1.1. Review of the Literature

A literature attempts to limit the multiplicity of equilibria in network markets by using equilibrium refinements. For example, it is sometimes assumed that agents are able coordinate to Pareto-superior equilibria (Hagiu (2006), Rasmusen (2007)). Such coordination may be facilitated when buyers make their decisions sequentially (Farrell and Saloner (1985), Rasmusen (2007)), or engage in cheap-talk communication before irrevocable decisions are made (Farrell and Rabin (1996)), or are in a position to form self-enforcing coalitions (Bernheim, Peleg and Whinston (1987)).[5] However, in practice, agent coordination may often be difficult to attain, especially when the number of agents is large. Other articles assume that agent expectations are *a priori* favorable to a specific focal brand; for example, such a brand may have reputational capital or a strong brand name (Caillaud and Jullien (2001, 2003), Hagiu (2006), Jullien (2011), Halaburda and Yehezkel (2016), Halaburda, Jullien and Yehezkel (2020)). However, although such an approach leads to useful insights in case it is plausible for a brand to be focal, the market outcome is still impacted by the somewhat arbitrary assumption about which specific

[4] As is well-known, Microsoft tended to sell its products through individualized agreements with computer manufacturers, which left ample room for price discrimination; an external price list often did not even exist (Baseman, Warren-Boulton and Woroch (1995)). Google also practiced price discrimination among device manufacturers (EC (2018)). In any case, our analysis does not have general implications for the social efficiency of such strategies (since competition does not necessarily lead to the social optimum). Social efficiency can only be determined by estimating the specific product parameters.

[5] Some articles discuss insulating tariffs (rather than equilibrium refinements), which aim to make prices to a certain degree contingent on the number of final users or on the extent of final network externalities. Insulating tariffs may contribute to the amelioration of coordination failures (e.g., Weyl (2010), White and Weyl (2016)). However, as Farrell and Klemperer (2007) point out, although contingent pricing seems theoretically promising, it is probably little used in practice. In any case, it is possible that the rise of artificial intelligence technology may facilitate the application of insulating tariffs in the future.

brand is favored.[6] We contribute to the literature by showing how the interaction between stand-alone and network benefits under price discrimination along with a rather mild refinement, the monotonicity criterion, may shape divide-and-conquer strategies, leading to a unique equilibrium. We bring out the concept of the dominant value margin, which may be a metric of the effectiveness of divide-and-conquer strategies.

Our paper supplements a line of research that discusses the utilization of divide-and-conquer strategies by a non-focal supplier that attempts to overcome its disadvantage vis-a-vis a focal competitor (which enjoys favorable *a priori* agent expectations). The ability of a non-focal supplier to price discriminate and to possibly divide and conquer constrains the equilibrium profits of the focal brand that must keep non-focal suppliers at bay (e.g., Caillaud and Jullien (2001, 2003), Jullien (2011)). We add to this research in two main ways. First, our analysis examines unrestrained supplier competition in that no supplier faces *a priori* impediments to its ability to compete. Players compete on an equal footing; there are no focal suppliers, and also all suppliers are able to set their prices simultaneously and sell to a large market.[7] Second, we place special emphasis on individual product features, considering differences in both stand-alone and network benefits across brands.[8] This allows us to bring out the disproportionate impact of stand-alone benefits on the effectiveness of divide-and-conquer strategies and to develop the important concept of the dominant value margin. Furthermore, we reveal a type of social inefficiencies that may occur when the socially inferior supplier has a stand-alone-benefit advantage, as well as a modest network-benefit disadvantage.

Some articles discuss how the interplay between stand-alone and network benefits may affect the multiplicity of equilibria (e.g., Halaburda, Jullien and Yehezkel (2020)). When two brands have the same network benefit in case they become dominant, and the

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[6] Chan (forthcoming) shows how potential maximization may constitute an equilibrium refinement. Potential maximization may be a useful and relevant criterion, especially when agents are significantly concerned about the possibility of deviation by other agents. However, it may be more idiosyncratic and less general than the monotonicity criterion that is used in the literature (e.g., Caillaud and Jullien (2001, 2003), Armstrong and Wright (2007)) and our analysis; potential maximization satisfies monotonicity, but monotonicity may not always satisfy potential maximization.

[7] In Jullien (2011), on the other hand, the focal supplier is a Stackelberg leader in prices. In Caillaud and Jullien (2001, 2003) both the focal and non-focal supplier set their prices simultaneously. However, in their standard one-sided-externalities models (to which their base two-sided models are equivalent when the two sides are identical) the battleground is rather small since suppliers compete for two customers only.

[8] In Caillaud and Jullien (2001, 2003), on the other hand, all suppliers are identical, while in Jullien (2011) suppliers differ only in their stand-alone benefits.

difference between the stand-alone benefits of the two brands is sufficiently larger than such a common network benefit, the brand that enjoys the stand-alone advantage always prevails in equilibrium. The reason is that price competition between the two brands approaches standard Bertrand competition (which, as is well-known, does not entail network externalities). In our paper the interplay between stand-alone and network benefits also plays an important role, but the reasons and the mechanics are different than the literature. In our analysis it is a weighted comparison between the two brands' *difference* in stand-alone benefits and their *difference* in network benefits that determines the winning brand. In such a comparison the difference in stand-alone benefits has a disproportionately large impact (or is assigned a disproportionate weight) because it has a disproportionate impact on the effectiveness of divide-and-conquer strategies (and on dominant value margins). This literature, on the other hand, abstracts from divide-and-conquer strategies. It examines a comparison between the two brands' difference in stand-alone benefits and their (common) *absolute* network benefit (rather than their difference in network benefits), which corresponds to the degree of proximity to standard Bertrand competition (rather than to an evaluation of divide-and-conquer strategies).

Furthermore, our paper brings out the possibility of a type of social inefficiencies that has not received sufficient attention in the literature on networks. In our analysis stand-alone benefits play a crucial role in the emergence of socially inefficient outcomes. A socially inferior brand prevails in equilibrium when it has a larger stand-alone benefit than a socially superior brand that merely enjoys a modest (rather than a large) advantage in network benefits. This stands in contrast to the concerns often raised in the literature that a technically inferior product on a stand-alone, as well as on a social welfare, basis may sometimes inefficiently prevail because of agent expectations or sell-fulfilling prophecies (e.g., David (1985), Farrell and Klemperer (2007), Halaburda and Yehezkel (2016), Halaburda, Jullien and Yehezkel (2020)).

In addition, the literature discusses various other sources of social inefficiencies. For example, a socially inefficient outcome may occur when a socially inefficient brand is focal through *a priori* favorable agent expectations (e.g., Caillaud and Jullien (2001, 2003), Hagiu (2006), Jullien (2011), Halaburda and Yehezkel (2016), Halaburda, Jullien and Yehezkel (2020)), or enjoys an incumbency advantage through the presence of customer switching costs (e.g., Farrell and Klemperer (2007)), ample customer migration opportunities (e.g., Biglaiser, Cremer and Vega (2021)) and variable feedbacks (e.g., Lamberson and Page (2018)). Sufficient customer heterogeneity may also give rise to socially inefficient outcomes (e.g., Jullien (2011), White and Weyl (2016), Biglaiser, Cremer and Vega (2021)). In our analysis suppliers compete on an equal footing with no focality or incumbency advantages. Furthermore, we focus on tippy network markets in that customers are homogeneous (e.g., Shapiro and Varian (1999)). Then, we supplement the literature by discussing another type of social inefficiencies; a stand-alone-benefit advantage (as well as a modest network-benefit disadvantage) of a socially inferior over a socially superior brand may be the driver of an inefficient outcome.

In a different vein, there is a line of research in contract theory that discusses possible divide-and-conquer strategies by a principal that attempts to elicit contract acceptance by several agents (e.g., Rasmusen, Ramseyer and Wiley (1991), Segal and Whinston (2000), Segal (2003), Halac, Kremer and Winter (2020)). For example, an incumbent upstream supplier may resort to divide-and-conquer strategies to impose long-term exclusive dealing contracts on downstream customers, aiming to block the future entry of potential rivals into the upstream market. Unlike the generic contract theory literature, our paper specifically examines network markets, modeling important idiosyncratic features of networks, such as the role of stand-alone and network benefits.[9] Furthermore, in our analysis all competing suppliers are able to adopt divide-and-conquer strategies, while in most of the contract theory literature such strategies are employed only by an incumbent monopolist or principal (that often aims to abuse its dominance).

2. THE MODEL

There are two suppliers, 1 and 2, that each attempts to sell its own brand of a product to a continuum of identical customers, whose measure is normalized to one. Our analysis focuses on tippy network markets that are susceptible to winner-takes-all outcomes; many important network products fall into this category. As Shapiro and Varian (1999), among others, point out, network markets tend to be tippy when

[9] For example, in the exclusive dealing literature the externality that downstream customers exert on each other is of a dual nature, i.e., accommodating or not the entry of a future upstream supplier. We, on the other hand, model explicit and continuous network benefits, rather than a dual all-or-nothing externality.

customers have sufficiently homogeneous tastes and preferences. Thus we assume that customers are identical. Furthermore, for expositional purposes, we assume a continuum of customers since it allows us to present our argument in a clear and straightforward manner. Our argument would be largely similar if the number of customers were finite; a continuum is a limit case as the number of customers approaches infinity.

For simplicity, it is assumed that both suppliers have zero production costs. In the appendix we will extend our results to incorporate marginal cost differences between suppliers. As is standard in the literature, a customer has a unit demand, consuming either one or zero units of the product. Each brand of the product entails network externalities. Thus the reservation value, $v_i(x_i)$, of brand i ($i \in \{1,2\}$) to a customer when a measure x_i of customers buy brand i is

$$v_i(x_i) = \alpha_i + n_i x_i , \tag{1}$$

where $\alpha_i > 0$ represents the stand-alone benefit of brand i and $n_i x_i$ ($n_i > 0$) corresponds to the network benefit. The two brands are incompatible in that the network benefits of each one depend only on the measure of its own customers, rather than of customers that buy the rival brand. For simplicity, the base model assumes that $2\alpha_2 \geq n_1 - n_2$ so that there is an interior solution regarding the upper bound of the prevailing supplier's profit. As the appendix will explain, our results are even stronger when $2\alpha_2 < n_1 - n_2$.

We define each supplier's $i \in \{1,2\}$ *dominant value margin* Δ_i as follows:

$$\Delta_i = v_i(1) - v_{-i}(0) = \alpha_i + n_i - \alpha_{-i} , \tag{2}$$

where $-i \neq i$. Specifically, in a possible winner-takes-all outcome Δ_i represents the difference between the social value (or the social surplus) of a sale to a customer by the fully dominant and the fully dominated brand when brand i is fully dominant, i.e., when all customers buy brand i ($x_i = 1$, $x_{-i} = 0$). Since in the base model suppliers have zero production costs, a sale's social value corresponds to the selling brand's reservation value to a customer.

Without any loss of generality, we assume that $\Delta_1 > \Delta_2$. Furthermore, to make the analysis of network externalities more straightforward, we assume that both suppliers are viable, i.e., $\Delta_2 > 0$, so that the ranking of the reservation values of the two brands (based on condition (1)) may depend on the measure of customers that buy each brand. This is an important feature of several network markets (e.g., Farrell and Saloner (2007)). Otherwise, supplier 2's nonviability ($\Delta_2 \le 0$) would have the rather extreme implication that even if all customers bought from supplier 2, allowing brand 2 to generate the largest possible network benefit, brand 1 would still be more valuable to customers solely on the basis of its stand-alone benefit. Then, supplier 2 would never be able to sell to any customers, and all customers would always buy from supplier 1 in equilibrium (e.g., Halaburda, Jullien and Yehezkel (2020)).

Suppliers are able to practice perfect price discrimination and sell to each customer at a different price. Thus each supplier i ($i \in \{1,2\}$) chooses a price $p_i(j) \in \mathbb{R}$ that it charges each customer j ($j \in [0,1]$) for a unit of its brand; a price may also be negative (constituting a subsidy). Then, customer j's surplus is $\alpha_i + n_i x_i - p_i(j)$ if the customer buys the product from supplier i. Section 4.2 will explain how our results may apply to imperfect price discrimination. Prices are observable by all agents. In section 4.3 we will discuss unobservable price offers. Furthermore, as is standard in many network games (e.g., Katz and Shapiro (1985), Caillaud and Jullien (2001, 2003), Farrell and Saloner (2007), Jullien (2011)), customers make their buying decisions simultaneously. In section 3.4 we will explain how our results carry through to sequential supplier offers.

We thus have the following two-stage game:

Stage 1: Each supplier i ($i \in \{1,2\}$) chooses a price $p_i(j) \in \mathbb{R}$ at which it offers a unit of its brand to each customer j ($j \in [0,1]$).

Stage 2: Each customer j ($j \in [0,1]$) chooses whether to accept a supplier's offer.

For simplicity, we adopt the tie-breaking convention that when a customer is indifferent between buying from supplier 1 and 2, it chooses to buy from supplier 1.

Overall, our model follows the standard game theory methodology of rational expectations and subgame perfection. Furthermore, we adopt a simple equilibrium refinement, the monotonicity criterion, which is akin to the standard property of monotonicity in social choice and political science (e.g., Moulin (1983), Arrow, Sen and Suzumura (2002)) and is also used in some articles in the literature on networks (e.g., Caillaud and Jullien (2001, 2003), Armstrong and Wright (2007)).

Suppose that in a subgame-perfect equilibrium a set S_i of customers buy from supplier $i \in \{1,2\}$, which offers each customer $j \in S_i$ a price $p_i(j)$. Then, the monotonicity criterion eliminates subgame-perfect equilibria in which the equilibrium strategy profiles include (at decision points outside the realized equilibrium path) price offers $p_i(j)' \le p_i(j)$, $\forall j \in S_i$ by one supplier $i \in \{1,2\}$ and $p_{-i}(j)' = p_{-i}(j)$, $\forall j \in [0,1]$ by the other supplier $-i \neq i$, as well as the presence of at least one customer in S_i that does not buy from supplier i after such price offers. In particular, according to the monotonicity criterion, a subgame-perfect equilibrium is eliminated when agents expect (based on the agent strategy profiles at decision points outside the realized equilibrium path) that a weak improvement in a supplier's price offers to all its equilibrium customers, while the competitor's prices remain constant, would lead to the rejection of the supplier's price offer by at least one equilibrium customer.[10] Such a criterion aims to eliminate rather unreasonable or frivolous equilibria in which customers change their product choices for no apparent reason, or in which an unanswered (by competitors) improvement in all price offers to its equilibrium customers by a supplier would (rather unreasonably) lead to a smaller measure of such customers buying from the supplier.

3. EQUILIBRIUM

We now solve for the equilibrium of the game. At first (section 3.1) we momentarily depart from the base model by assuming that suppliers are unable to price discriminate. Then, in sections 3.2, 3.3 and 3.4 we discuss in detail several aspects of the

[10] No subgame-perfect equilibria are eliminated on the basis of the prices $p_i(j)'$ that supplier i may offer to customers $j \in S_{-i}$ other than its equilibrium customers, or on the basis of the actions of such customers (since they do not buy from i anyway in equilibrium). The monotonicity criterion is mild and only eliminates equilibria with rather unreasonable responses of equilibrium customers to improved offers.

equilibrium of the base model in which suppliers have the ability to practice price discrimination. By examining uniform pricing in addition to the base model, we can bring out clearly the crucial role of price discrimination in equilibrium outcomes.

3.1. Uniform Pricing

In this section we momentarily assume that each supplier i ($i \in \{1,2\}$) offers all customers a uniform price, i.e., $p_i(j) = p_i$, $\forall j \in [0,1]$; suppliers are unable to price discriminate. Then, given that the network market is tippy, and both suppliers are viable ($\Delta_i > 0$, $i \in \{1,2\}$), there are two possible equilibrium outcomes regarding market shares in which either supplier $i \in \{1,2\}$ may sell to all customers, i.e., either $x_1{**} = 1$ or $x_2{**} = 1$.[11] In each equilibrium outcome there is a range of possible equilibrium prices by winning supplier i ($i \in \{1,2\}$), i.e., if $x_i{**} = 1$, we have $0 \le p_i{**} \le \Delta_i$. Furthermore, there are several possible prices by losing supplier $-i \ne i$ (which does not sell to any customers anyway) that support such an equilibrium outcome (also depending on customer expectations). To economize on notation, we assume that losing supplier $-i$ chooses the most competitive or the lowest price $p_{-1} = p_i - \Delta_i$ for which winning supplier i still sells to customers. Then, all customers marginally decide to buy from supplier i (because of the tie-breaker).[12] Finally, the monotonicity criterion does not affect agent strategies in the uniform pricing game (although, as we will see, it is relevant under price discrimination).

The existence of multiple equilibria, — or, specifically, of two winner-takes-all equilibria, — under uniform pricing is hardly a surprising result since it is consistent with the well-known notion that network markets may give rise to multiple equilibrium outcomes (e.g., Katz and Shapiro (1985), Shapiro and Varian (1999), Farrell and Saloner

[11] There can be no equilibrium in which both suppliers sell to a strictly positive measure of customers. Suppose, for example, that suppliers i and $-i$ ($i \ne -i$) profitably sell to a measure $x_i > 0$ and $x_{-i} > 0$ of customers, respectively ($p_i \ge 0$, $p_{-i} \ge 0$). Then, if $\alpha_i + n_i x_i - p_i \ge \alpha_{-i} + n_{-i} x_{-i} - p_{-i}$ ($\alpha_i + n_i x_i - p_i \le \alpha_{-i} + n_{-i} x_{-i} - p_{-i}$), the customers of supplier $-i$ (supplier i) would have an incentive to deviate and buy from supplier i (supplier $-i$).

[12] To simplify the description of an equilibrium with $x_2{**} = 1$, we assume that in such an equilibrium when a customer is indifferent between buying from supplier 1 and supplier 2, it decides to buy from supplier 2, i.e., (only in such a case) we reverse the tie-breaking convention of section 2.

(2007), Jullien (2011), Halaburda, Jullien and Yehezkel (2020)). Section 3.2 will show how the capability of suppliers to practice price discrimination may lead to a unique outcome regarding the identity of the prevailing supplier. Proposition 1 follows.

Proposition 1: If suppliers practice uniform pricing, there exist two subgame-perfect equilibria regarding market shares, i.e., $x_i^{**}=1$, $i\in\{1,2\}$. Such equilibria entail prices $0\le p_i^{**}\le\Delta_i$ and $p_{-i}^{**}=p_i^{**}-\Delta_i$. Supplier i's equilibrium profit is $0\le\Pi_i^{**}\le\Delta_i$.

Proof: It follows directly from the discussion above.

Intuitively, as is well-known, in a network market a customer's buying decision may depend on the buying decisions of all other customers since such decisions affect the reservation values of the various brands. Thus in our analysis there are two subgame-perfect equilibria regarding market shares, and such equilibria entail winner-takes-all outcomes. Each customer buys from supplier i ($i\in\{1,2\}$) in equilibrium since all other customers also buy from i.

3.2. Perfect Price Discrimination

We now examine the equilibrium of the base model of section 2 where suppliers are able to practice perfect price discrimination (should they deem suitable). Then, in all subgame-perfect equilibria that meet the monotonicity criterion supplier 1 always sells to all customers, i.e., $x_1^*=1$ and $x_2^*=0$, exploiting its larger dominant value margin (unlike the multiple equilibrium market shares under uniform pricing in proposition 1). In particular, as we show in the appendix, there exist no equilibria in which supplier 2 sells to a strictly positive measure of customers $S_2=[0,x_2]$ with $x_2\in(0,1]$ and $\int_0^{x_2} p_2(j)dj\ge 0$ (so that supplier 2's pricing strategy is weakly profitable). Such an equilibrium cannot stem from the equilibrium strategy profiles of suppliers 1 and 2 because it can always be profitably countered by supplier 1 in stage 1. Given the prices of supplier 2, supplier 1 can successfully make divide-and-conquer price offers to customers in S_2 (also knowing that according to the monotonicity criterion, it will continue to sell to the measure $S_1 = (x_2, 1]$ of its own customers after poaching customers in S_2), exploiting its larger dominant value margin. For example, supplier 1 may offer each customer $j \in S_2 = [0, x_2]$ a price $p_2(j) + v_1(1 - x_2 + j) - v_2(x_2 - j)$ in stage 1; each customer $j \in [0, x_2]$ would then buy from supplier 1 in stage 2 since customers $j' < j$ would simultaneously always also buy from supplier 1 (and regardless of the buying decisions of customers $j'' > j$).[13]

<<FIGURE 1 HERE>>

We may employ geometry to better understand the intuition behind supplier 1's counter. Suppose that in a hypothetical equilibrium all customers bought from supplier 2. Even in the face of such an adverse expected outcome, supplier 1 would be able to capture some customers by offering them sufficiently lower prices than supplier 2. As figure 1 shows, for the most extreme of those negative-price-differential customers the absolute value of the negative price difference between supplier 1 and supplier 2 is close to supplier 2's dominant value margin Δ_2 (since such customers must be convinced to buy from supplier 1 irrespective of the actions of most other customers). However, supplier 1 does not need to make so low price offers to all customers. Each customer that (on the basis of price offers) is bound to be captured by supplier 1 automatically increases the reservation value of supplier 1 relative to that of supplier 2 in the eyes of other customers. Then, in stage 1 supplier 1 is able to make progressively increasing simultaneous price offers relative to supplier 2, rationally expecting that it will capture such customers in stage 2. It follows that supplier 1 also poaches some customers by charging them substantially higher prices than supplier 2. For the most extreme of those positive-price-differential customers the positive price difference between suppliers 1 and 2 is close to supplier 1's dominant value margin Δ_1 (since most other customers are bound to be captured by supplier 1). Given that $\Delta_1 > \Delta_2$, positive-price-differential

[13] As explained above, the monotonicity criterion is an important element in supplier 1's counter when $x_2 \in (0,1)$. If, however, $x_2 = 1$, the monotonicity criterion does not affect supplier 1's counter (since S_1 is empty). Overall, without the monotonicity criterion suppliers' ability to price discriminate could generate multiple interior (and rather unreasonable) subgame-perfect equilibria. The monotonicity criterion eliminates such unreasonable equilibria.

customers more than compensate for negative-price-differential customers, so that such a possible counter by supplier 1 is always profitable (see figure 1).

Furthermore, since $\Delta_1 > \Delta_2$, supplier 2 is less able to implement such divide-and-conquer poaching strategies in subgames where competing supplier 1 sells to a strictly positive measure of customers. To simplify the notation, suppose that in equilibrium supplier 1 offers prices to customers $j \in [0,1]$ in weakly descending order (i.e., $p_1(j') \geq p_1(j'')$ if $j' < j''$). Then, as we show in the appendix, profitable poaching by supplier 2 is not possible if supplier 1's prices are sufficiently low so that for any $z \in [0,1]$, the following condition is met:

$$0 \leq \int_0^z p_1(j)^* dj \leq \int_0^z [v_1(1-j) - v_2(j)]dj = z[\Delta_1 - \frac{z(n_1 + n_2)}{2}], \ \forall z \in [0,1] \ . \qquad (3)$$

Given supplier 1's prices are sufficiently low to meet condition (3) and the expectation that all customers will buy from supplier 1, supplier 2 is unable to earn a weakly positive profit by poaching any group $z \in [0,1]$ of supplier 1's most lucrative customers that can be more easily tempted to buy from supplier 2. Multiple possible equilibrium prices by supplier 1 exist so that condition (3) is met.[14]

Since supplier 2 does not sell to any customers in equilibrium, and given supplier 1's prices, several possible prices $p_2(j)$, $\forall j$ by supplier 2 would support such an equilibrium outcome (also depending on agent expectations). To economize on notation, we assume that supplier 2 chooses the most competitive or the lowest price $p_2(j)^* = p_1(j)^* - \Delta_1$ for which supplier 1 still sells to customer j. Then, all customers marginally decide to buy from supplier 1 (because of the tie-breaker). Supplier 1's profit in such a subgame-perfect equilibrium is $0 \leq \Pi_1^* = \int_0^1 p_1^*(j)dj \leq \frac{\Delta_1 - \Delta_2}{2}$. Π_1^* depends on the degree of the price squeeze by supplier 2 since supplier 1 exploits to varying degrees its larger dominant value margin. We can also see that equilibrium

[14] For example, supplier 1 may offer a uniform price $p_1(j)$ so that $0 \leq p_1(j) \leq \frac{\Delta_1 - \Delta_2}{2}$, $\forall j \in [0,1]$.

prices (see, for example the uniform prices of note 14) may not always entail actual price discrimination (although suppliers' ability to practice price discrimination in their strategy profiles outside the realized equilibrium path is crucial to our results). Proposition 2 follows.

Proposition 2: In all subgame-perfect equilibria that meet the monotonicity criterion supplier 1 sells to all customers, i.e., $x_1{*}=1$, $x_2{*}=0$. Such equilibria exist and entail prices $0 \le \int_0^z p_1(j)^* dj \le \int_0^z [v_1(1-j) - v_2(j)]dj = z[\Delta_1 - \frac{z(n_1+n_2)}{2}]$, $\forall z \in [0,1]$ and $p_2(j)^* = p_1(j)^* - \Delta_1$, $\forall j \in [0,1]$. Supplier 1's equilibrium profit is $0 \le \Pi_1{*} \le \frac{\Delta_1 - \Delta_2}{2}$.

Proof: The proof is in the appendix.

Intuitively, price discrimination allows supplier 1 to prevent any sales to customers by supplier 2 through the adoption of divide-and-conquer pricing if necessary. In such a poaching scheme supplier 1 may attract some customers by offering them negative price differentials relative to supplier 2, which in turn enables supplier 1 to sell to the remaining customers at positive price differentials. Dominant value margins are a metric of the effectiveness of divide-and-conquer strategies; enjoying an advantage in divide-and-conquer poaching, the supplier with the larger dominant value margin, i.e., supplier 1, always sells to all customers in equilibrium. In supplier 1's divide-and-conquer poaching strategies positive-price-differential customers more than compensate for negative-price-differential customers, giving supplier 1 the ability to profitably divide and conquer and thus to always prevail in equilibrium.

Furthermore, we can see that stand-alone benefits, α_1 and α_2, have a disproportionately large, or a double, impact on the difference between dominant value margins ($\Delta_1 - \Delta_2 = 2(\alpha_1 - \alpha_2) + (n_1 - n_2)$), — and thus on the effectiveness of divide-and-conquer strategies, — since they are generated by both a fully dominant and a fully dominated brand. Network benefits, n_1 and n_2, on the other hand, are generated only by a fully dominant brand (which is actually sold to customers). Then, if a supplier i has a specific reservation value $\bar{v} = v_i(1) = \alpha_i + n_i$ in case it sells to the entire market, it is more difficult *ceteris paribus* for the competing supplier $-i$ to employ divide-and-conquer poaching strategies when i's stand-alone benefit α_i constitutes a larger fraction of $\bar{v}$. As the competitor $-i$ poaches some of i's customers, the remaining customers do not become much easier to poach since i's reservation value does not fall by much. Similarly, supplier i's divide-and-conquer poaching strategy against $-i$ is facilitated.

3.3. Social Welfare

Social welfare is the sum of costumer surplus and supplier profits. Since prices constitute a mere transfer from customers to suppliers, and suppliers have zero production costs, social welfare is equal to the total reservation value of the products that customers buy. Furthermore, since the network market is tippy, social welfare is never maximized in interior outcomes where both suppliers sell to a strictly positive measure of customers.[15] Thus the social optimum is determined by a comparison of the two winner-takes-all outcomes in which supplier i ($i \in \{1,2\}$) sells to all customers. In this regard we define the dominant social value of supplier i ($i \in \{1,2\}$) as the social value (or social surplus) per customer in case supplier i sells to all customers (which in the base model corresponds to brand i's reservation value at $x_i = 1$ since suppliers have zero costs). In particular, the dominant social value of supplier i ($i \in \{1,2\}$) is $v_i(1) = \alpha_i + n_i$. When $v_i(1) > v_{-i}(1)$ ($-i \neq i$) supplier i has a larger dominant social value than its competitor $-i$, and the social optimum occurs when supplier i sells to all customers.

In all subgame-perfect equilibria that meet the monotonicity criterion supplier 1 sells to all customers (proposition 2). Equilibrium social welfare thus is $v_1(1) = \alpha_1 + n_1$. Such an equilibrium outcome is socially optimal when $v_1(1) - v_2(1) = (\alpha_1 + n_1) - (\alpha_2 + n_2) \geq 0$, i.e., when supplier 1 has a larger dominant social

[15] Suppose, for example, that suppliers i and $-i$ ($i \neq -i$) sell to a measure $x_i > 0$ and $x_{-i} > 0$ of customers, respectively. Then, if $\alpha_i + n_i x_i \geq \alpha_{-i} + n_{-i} x_{-i}$ ($\alpha_i + n_i x_i \leq \alpha_{-i} + n_{-i} x_{-i}$), social welfare strictly

value than supplier 2. However, since in equilibrium the supplier with the larger dominant value margin, rather than the larger dominant social value, sells to all customers, the equilibrium outcome is not necessarily socially optimal. In particular, when supplier 2 has a larger network benefit ($n_2 > n_1$), and supplier 1 has a larger stand-alone benefit, but not too large to make supplier 1 more efficient ($\frac{n_2 - n_1}{2} < \alpha_1 - \alpha_2 < n_2 - n_1$), supplier 1 sells to all customers in equilibrium (since $\Delta_1 > \Delta_2$), although supplier 2 would generate more social welfare if it were the prevailing supplier ($v_1(1) < v_2(1)$). Then, the equilibrium outcome fails to be socially optimal. For all parameters outside this range the game leads to the social optimum. Proposition 3 follows.

<u>Proposition 3:</u> In any subgame-perfect equilibrium that meets the monotonicity criterion the outcome of the game fails to be socially optimal if $n_2 > n_1$ and $\frac{n_2 - n_1}{2} < \alpha_1 - \alpha_2 < n_2 - n_1$. It is socially optimal for all parameters outside this range.

Proof: The proof is in the appendix.

Intuitively, in proposition 2 a comparison of dominant value margins, rather than of dominant social values, determines the supplier that prevails in the game. Thus in equilibrium supplier 1 may sometimes sell to all customers even when supplier 2 is more efficient in that supplier 2 has a larger dominant social value. Such a socially inefficient outcome may occur when the socially inferior supplier has an advantage in stand-alone benefits and a modest disadvantage in network benefits (that is nonetheless insufficient to prevent its prevalence). Then, given the disproportionate impact of stand-alone benefits on dominant value margins, the socially inferior supplier takes full advantage of its larger stand-alone benefit and prevails in equilibrium.

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increases if, for example, the customers of supplier $-i$ (supplier i) deviate and buy from supplier

3.4. Sequential Supplier Offers

In the base model suppliers $i \in \{1,2\}$ set their prices $p_i(j)$ ($j \in [0,1]$) simultaneously in stage 1, while all customers whether to accept such offers simultaneously in stage 2. However, our basic results largely carry through when suppliers interact with customers sequentially as long as agents expect that the degree to which a dominant supplier is able to exploit its dominant value margin (or, the degree of the price squeeze by the dominated supplier) is similar regardless of whether supplier 1 or 2 becomes dominant. In the sequential game suppliers 1 and 2 interact with customers $j \in [0,1]$ in ascending order, starting with customer $j = 0$ and concluding with customer $j = 1$. Suppliers 1 and 2 simultaneously offer customer j ($j \in [0,1]$) their prices $p_i(j)$ ($i \in \{1,2\}$) after all previous customers $j' < j$ have decided whether to accept suppliers' offers. Customer j chooses whether to accept a supplier's offer, and then suppliers 1 and 2 proceed to offer prices to the next customer. Prices and customer decisions are observable by all agents. Such sequential interaction between suppliers and customers is in the spirit of articles in the contract theory literature (e.g., Rasmusen, Ramseyer and Wiley (1991), Segal and Whinston (2000)). Furthermore, assume for the moment that when a supplier expects that it is unable to win any subsequent customers in the sequential game, it does not charge such customers a negative price, i.e., the degree, ρ, of its price squeeze is zero.[16]

As we explain in the appendix on the proof of proposition 4, if supplier 1 (supplier 2) has sold to a measure $\overline{x_1}$ ($\overline{x_2} = 1 - \overline{x_1}$) of customers, the reservation value of supplier 1's (supplier 2's) brand is weakly larger even if all the remaining measure $1 - \overline{x_1}$ ($1 - \overline{x_2}$) of customers buy from supplier 2 (supplier 1). Since $\Delta_1 > \Delta_2$, we have $\overline{x_1} < 0.5 < \overline{x_2} = 1 - \overline{x_1}$. Then, if supplier 1 has sold to a measure $\overline{x_1}$ of customers, it will always sell to all subsequent customers. Furthermore, suppose that in a subgame suppliers 1 and 2 have sold to a measure x_1 and x_2 of customers, respectively, where

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i (supplier $-i$). Thus $x_i > 0$, $x_{-i} > 0$ is never the social optimum.

[16] For example, a similar assumption is often made in standard Bertrand simultaneous price games in which two competing firms with different unit costs sell a homogeneous product. Then, it is often assumed that the losing firm (with the higher unit cost) refrains from setting a price strictly lower than its unit cost.

$\overline{x_1} - x_1 < \overline{x_2} - x_2$ (or $\overline{x_1} - \overline{x_2} < x_1 - x_2$). As we explain in the appendix, backward induction implies that all customers $j \in (x_1 + x_2, 1]$ buy from supplier 1. We can also see that for customer $j = 0$ we have $x_1 = x_2 = 0$ and thus $\overline{x_1} - x_1 < \overline{x_2} - x_2$ (since $\Delta_1 > \Delta_2$). Thus according to backward induction, in a subgame-perfect equilibrium all customers $j \in [0,1]$ always buy from supplier 1 that has a larger dominant value margin.[17] Since the degree of price squeeze is zero, supplier 1 sets a price $p_1(j) = \Delta_1$, $\forall j \in [0,1]$ (while $p_2(j) = 0$). In addition, our results carry through to any degree of price squeeze $\rho \in [0,1]$ (rather than only to $\rho = 0$ as above) by the losing supplier. Then, given that $\Delta_1 > \Delta_2$, in a subgame-perfect equilibrium all customers $j \in [0,1]$ always buy from supplier 1 at a price $p_1(j) = (1-\rho)\Delta_1$, $\forall j \in [0,1]$. Finally, the social welfare analysis is identical to the base model (proposition 3). We summarize in proposition 4.

Proposition 4: In all subgame-perfect equilibria of the sequential offers game supplier 1 sells to all customers, i.e., $x_1{*} = 1$, $x_2{*} = 0$. Such equilibria exist $\forall \rho \in [0,1]$ and entail prices $p_1(j){*} = (1-\rho)\Delta_1$, $\forall j \in [0,1]$, while $p_2(j){*} = p_1(j){*} - \Delta_1$, $\forall j \in [0,1]$. Supplier 1's equilibrium profit is $0 \le \Pi_1{*} = (1-\rho)\Delta_1 \le \Delta_1$. Proposition 3 still carries through.

Proof: The proof is in the appendix.

Intuitively, sequential offers by suppliers strengthen supplier 1's ability to always prevail in the market. The upper bound, Δ_1, of supplier 1's profit in proposition 4 is higher than its upper bound, $\frac{\Delta_1 - \Delta_2}{2}$, in the base model of simultaneous offers (proposition 2).[18] In particular, since $\Delta_1 > \Delta_2$, or $\overline{x_1} < \overline{x_2}$, no customer in the sequential

[17] The monotonicity criterion, which refines simultaneous customer decisions, is not applicable to the sequential game.

[18] In both the simultaneous and the sequential offers game the equilibrium profit of supplier 1 lies in a range (propositions 2 and 4) and depends on the degree of price squeeze by losing supplier 2 since supplier 1 exploits to varying degrees its larger dominant value margin. As a result, a comparison between supplier 1's equilibrium profits in the two games is not meaningful. However, we can see that the upper bound of

process is pivotal; supplier 1 will sell to all subsequent customers anyway regardless of a customer's buying decisions. Thus in the most lucrative (on the part of supplier 1) subgame-perfect equilibrium supplier 1 is even able to extract a full winner-takes-all rent from each customer, i.e., charge each customer $j \in [0,1]$ a price Δ_1. Overall, there are multiple possible equilibrium prices by prevailing supplier 1 that depend on the degree, ρ ($\rho \in [0,1]$), of price squeeze by competing supplier 2, i.e., $p_1(j)^* = (1-\rho)\Delta_1$, $\forall j \in [0,1]$.

4. ROBUSTNESS

In this section we examine how our results may carry through to markets with more than two suppliers, imperfect price discrimination and unobservable price offers. Furthermore, we in the appendix we discuss how our results may carry through to different unit costs, non-linear network benefits and the case in which $2\alpha_2 < n_1 - n_2$.

4.1. More than Two Suppliers

In the base model there are two suppliers, 1 and 2. It is straightforward to extend the results to a game in which there are $m > 2$ suppliers. Suppose that the reservation value of the brand of supplier i is $v_i(x_i) = \alpha_i + n_i x_i$, $i \in \{1,...,m\}$, where $\alpha_i > 0$, $n_i > 0$, and x_i is the measure of customers that buy the product from supplier i. It is obvious that dominant value margins are transitive. In a pairwise comparison if supplier 1 has a larger dominant value margin than supplier 2, i.e., $2(\alpha_1 - \alpha_2) + n_1 - n_2 > 0$, and supplier 2 has a larger dominant value margin than supplier 3, i.e., $2(\alpha_2 - \alpha_3) + n_2 - n_3 > 0$, then supplier 1 has an even larger dominant value margin than supplier 3, i.e., $2(\alpha_1 - \alpha_3) + n_1 - n_3 > 2(\alpha_1 - \alpha_2) + n_1 - n_2 > 0$. Such transitivity implies that suppliers can be ranked on the basis of their dominant value margins. Then, the supplier in first place in the ranking, — such as supplier 1 above, — always sells to all customers in all subgame-perfect equilibria that meet the monotonicity criterion. The upper bound of the

supplier 1's equilibrium profit is higher in the sequential offers game (while the lower bound, 0, is the same in both games). It follows that supplier 1 has a higher potential for profit in the sequential offers game.

winning supplier's profit depends on the pairwise dominant-value-margin comparison between the winner and the runner-up, i.e., $0 \le \Pi_1{*} \le \frac{\Delta_1 - \Delta_2}{2}$.

4.2. Imperfect Price Discrimination

In the base model suppliers are able to make discriminatory price offers, which amount to perfect price discrimination. Our analysis, however, still holds if suppliers' ability to price discriminate is limited in that a supplier is able to charge each of N ($N \ge 2$) customer groups, rather than each individual customer, a different price; a supplier charges a uniform price within each of the N customer groups. For simplicity, we assume that all customer groups are the same size so that each group has a measure $1/N$ of customers. Our results carry through as long as supplier 1 has a sufficiently larger dominant value margin than supplier 2, — i.e., $\Delta_1 - \Delta_2$ is sufficiently large, — and price discrimination is not too imperfect, — i.e., N is not too small.

We can reexamine supplier 1's poaching strategy when it is able to price discriminate between N customer groups. In particular, suppose that there exists an equilibrium in which all customers buy from supplier 2 ($x_2 = 1$) and $\int_0^1 p_2(j)dj = 0$, which is the most difficult situation for supplier 1 to apply a counter (since in part A of the proof of proposition 1 x_2 is the largest ($x_2 = 1$) and $\int_0^{x_2} p_2(j)dj$ is the lowest ($\int_0^{x_2} p_2(j)dj = 0$)). Then, after adapting its base poaching strategy (condition (A1)) to group price discrimination, supplier 1 earns a profit $\int_0^1 p_2(j)dj + \sum_{k=0}^{N-1}[(\alpha_1 - \alpha_2) - (1 - \frac{k}{N})n_2 + \frac{k}{N}n_1]\frac{1}{N}$, which is equal to $\alpha_1 - \alpha_2 + \frac{n_1 - n_2}{2} - \frac{n_1 + n_2}{2N} = \frac{\Delta_1 - \Delta_2}{2} - \frac{n_1 + n_2}{2N}$. As long as $\Delta_1 - \Delta_2$ is sufficiently large and N is not too small, such a profit is positive, and supplier 1's poaching strategy is profitable.[19]

[19] We can see that as $N \to \infty$, imperfect price discrimination approaches first-degree discrimination and the base model exactly applies.

In particular, as in the base model, each captured customer automatically increases the reservation value of supplier 1 relative to that of supplier 2. However, unlike the base model, supplier 1 is unable to fully capitalize on such a progressive increase in reservation values through individualized divide-and-conquer strategies since it must offer a uniform price within each of the N customer groups. In any case if N is not too small and $\Delta_1 - \Delta_2$ is sufficiently large, the practice of group, rather than of perfect, price discrimination is sufficient to preclude the existence of an equilibrium in which supplier 2 sells to all customers (or to a strictly positive measure of customers for that matter). Supplier 1 always sells to all customers in all subgame-perfect equilibria that meet the monotonicity criterion. In addition, not only does supplier 1 always sell to all customers, but the upper bound of supplier 1's profit is also higher than in proposition 2 since the ability of supplier 2 to counter supplier 1 is hindered (given its inability to practice first-degree price discrimination).

Finally, if suppliers are totally unable to apply price discrimination and need to practice uniform pricing in the entire group of customers ($N = 1$), the poaching strategies of the base model are completely hindered. Condition $\frac{\Delta_1 - \Delta_2}{2} - \frac{n_1 + n_2}{2N}$ in the above paragraphs is equal to $2(-\alpha_2 - n_2 + \alpha_1) = -2\Delta_2 < 0$. Thus proposition 1 holds, and either supplier may sell to all customers in a subgame-perfect equilibrium.

4.3. Unobservable Price Offers

In the base model supplier price offers $p_i(j)$, ($i \in \{1,2\}$, $j \in [0,1]$) are observable by all agents. In this section we examine unobservable price offers; each customer is unable to observe the prices that are offered to other customers. Then, we can see that when price offers are sequential (section 3.4), our results directly carry through to unobservable prices. Since a customer $j \in [0,1]$ is able to observe the decisions of previous customers $j' < j$, the game of section 3.4 is unchanged; observing the specific prices that customers $j' < j$ were offered is immaterial.

We can also incorporate unobservable price offers into the base model with simultaneous customer decisions; price offers are unobservable, although (as is standard in games with unobservable offers) customers are able to observe each other's expectations about the measure of customers that will accept suppliers' offers. Then, price unobservability hinders the benchmark divide-and-conquer poaching strategies. For example, in the base model if supplier 2 sold to a strictly positive measure $x_2 \in (0,1]$ of customers, supplier 1 could poach such customers by offering each customer $j \in [0, x_2]$ a price $p_2(j) + v_1(1 - x_2 + j) - v_2(x_2 - j)$; a customer $j \in [0, x_2]$ would buy from supplier 1 since it could see (given the observable prices) that customers $j' < j$ were also bound to buy from supplier 1. However, if prices are unobservable, such reasoning does not carry through since customers are unable to infer from (the now unobservable) prices that customers $j' < j$ will buy from supplier 1. Similarly, supplier 2 is unable to utilize the base poaching strategies. Then, either supplier $i \in \{1,2\}$ may prevail in a winner-takes-all outcome. In such equilibria a prevailing supplier's price $p_i(j)$ ($i \in \{1,2\}$, $j \in [0,1]$) is never strictly negative because supplier i would incur a loss by selling to j without possibly deriving any benefits from increasing the expected network benefits of the remaining customers; since price offers are unobservable, the remaining customers expect j to buy from supplier i anyway in such equilibria.

In any case our results carry through to unobservable offers as long as supplier 1 has the ability unilaterally to make its price offers observable (while the competitor's prices, whose observability supplier 1 cannot affect, may still remain unobservable). Then, equilibria in which a strictly positive measure $x_2 \in (0,1]$ of customers buy from supplier 2 cannot stem from the equilibrium strategy profiles of suppliers 1 and 2 since they can always be profitably countered by supplier 1 in stage 1. In particular, since agents rationally understand that $p_2(j) \geq 0$, $\forall j \in [0, x_2]$, supplier 1 may offer each customer $j \in [0, x_2]$ a price $v_1(1 - x_2 + j) - v_2(x_2 - j)$ and unilaterally make its price offers observable. Each customer $j \in [0, x_2]$ would then buy from supplier 1 even if it had been offered the lowest possible price $p_2(j) = 0$ by supplier 2 because it could infer from (the unilaterally observable) prices that customers $j' < j$ would also buy from supplier 1. As in the base model, such a counter would always be profitable for supplier 1. Thus unilateral observability allows suppliers to implement poaching strategies similar to the base model (where observability is universal).[20] Supplier 1 always sells to all customers in all subgame-perfect equilibria that meet the monotonicity criterion.[21]

5. CONCLUSION

The pervasive role of self-fulfilling agent expectations often leads to multiple equilibria in network competition; such randomness makes difficult the drawing of solid conclusions about business strategy and antitrust intervention. Our analysis shows that tippy network markets, which are susceptible to winner-takes-all outcomes, may have a unique subgame-perfect equilibrium regarding the winning supplier. When first, as is often the case in practice, suppliers are able to practice price discrimination, and second, a mild equilibrium refinement, the monotonicity criterion, is adopted, network competition may generate a unique equilibrium regarding the winner's identity. Such equilibrium uniqueness stems from the ability of suppliers to utilize divide-and-conquer strategies and implies that the prevailing brand may be fully determined by its product features. We introduce the concept of the dominant value margin, which may be a metric of the effectiveness of divide-and-conquer strategies; the supplier with the larger dominant value margin may always sell to all customers in equilibrium. Such a market outcome is not always socially efficient since a socially superior supplier may fail to prevail if it has a stand-alone-benefit disadvantage and only a modest network-benefit advantage compared with its competitor.

[20] Similar to the base model poaching and the related unilateral observability do not occur on the realized equilibrium path. On the equilibrium path supplier 1 may keep its prices either unobservable or observable.
[21] The monotonicity criterion is unnecessary in subgames where all price offers are unobservable. The criterion only applies to observable prices. Then, similar to the base model, a subgame-perfect equilibrium is eliminated when an agent expects that a weak improvement in a supplier's observable price offers to all

REFERENCES

Armstrong, Mark, and Julian Wright. "Two-Sided Markets, Competitive Bottlenecks, and Exclusive Contracts," *Economic Theory*, 32, August 2007, p. 353-380.

Arrow, Kenneth J., Amartya K. Sen, and Kotaro Suzumura. *Handbook of Social Choice and Welfare*, Vol. 1, Amsterdam: North-Holand, 2002.

Baseman Kenneth C., Frederick R. Warren-Boulton, and Glenn A. Woroch. "Microsoft Plays Hardball: The Use of Exclusionary Pricing and Technical Incompatibility to Maintain Monopoly Power in Markets for Operating System Software," *The Antitrust Bulletin*, 40, Summer 1995, p. 265-315.

Bernheim, Douglas B., Bezalel Peleg, and Michael D. Whinston. "Coalition-Proof Nash Equilibria I: Concepts," *Journal of Economic Theory*, 42, June 1987, p. 1-12.

Biglaiser, Gary, Jacques Cremer, and Andre Vega. "Should I Stay or Should I Go? Migrating Away from the Incumbent Platform," Working Paper, Department of Economics, University of North Carolina, January 2021.

Caillaud, Bernard, and Bruno Jullien. "Competing Cybermediaries," *European Economic Review*, 45 (4-6), May 2001, p. 797-808.

Caillaud, Bernard, and Bruno Jullien. "Chicken & Egg: Competition Among Intermediation Service Providers," *RAND Journal of Economics*, 34 (2), Summer 2003, p. 309-328.

Chan, Lester T. "Divide and Conquer in Two-Sided Markets: A Potential-Game Approach," *RAND Journal of Economics*, forthcoming.

Cusumano, Michael A., Yiorgos Mylonadis, and Richard S. Rosenbloom. "Strategic Maneuvering and Mass-Market Dynamics: The Triumph of VHS over Beta," *Business History Review*, 66 (1), Spring 1992, p. 51-94.

David, Paul A. "Clio and the Economics of QWERTY," *American Economic Review*, 75 (2), May 1985, p. 332-337.

European Commission – Fact Sheet. *Antitrust: Commission Opens Formal Investigation against Google in Relation to Android Mobile Operating System*, Memo/15/4782, April 15, 2015.

European Commission – Press Release. Antitrust: Commission Fines Google €4.34 Billion for Illegal Practices Regarding Android Mobile Devices to Strengthen Dominance of Google's Search Engine, IP/18/4581, July 18, 2018.

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its equilibrium customers, while the competitor's prices remain either constant or unobservable, would lead to the rejection of the supplier's price offer by at least one equilibrium customer.

Farrell, Joseph, and Paul Klemperer. "Coordination and Lock-in: Competition with Switching Costs and Network Effects," in Mark Armstrong and R. Porter (eds.), Handbook of Industrial Organization, Vol. 3, North-Holland, 2007, p. 1969-2072.

Farrell, Joseph, and Matthew Rabin. "Cheap Talk," *Journal of Economic Perspectives*, 10 (3), Summer 1996, p. 103-118.

Farrell, Joseph, and Garth Saloner. "Standardization, Compatibility and Innovation," *RAND Journal of Economics*, 16 (1), Spring 1985, p. 70-83.

Farrell, Joseph, and Garth Saloner. "Installed Base and Compatibility: Innovation, Product Preannouncements, and Predation," *American Economic Review*, 76 (5), December 1986, p. 940-955.

Hagiu, Andrei. "Pricing and Commitment in Two-Sided Platforms," *RAND Journal of Economics*, 37 (3), Autumn 2006, p. 720-737.

Halaburda, Hanna, Bruno Jullien, and Yaron Yehezkel. "Dynamic Competition with Network Externalites: Why History Matters," *RAND Journal of Economics*, 51, Spring 2020, p. 3-31.

Halaburda, Hanna, and Yaron Yehezkel. "The Role of Coordination Bias in Platform Competition," *Journal of Economics and Management Strategy*, 25 (2), Summer 2016, p. 274-312.

Halac, Marina, Ilan Kremer, and Eyal Winter. "Raising Capital from Heterogeneous Investors," *American Economic Review*, 110 (3), March 2020, p. 889 – 921.

Jullien, Bruno. "Competition in Multi-Sided Markets: Divide and Conquer," *American Economic Journal: Microeconomics*, 2, November 2011, p. 186-219.

Katz, Michael L., and Carl Shapiro. "Network Externalities, Competition, and Compatibility," *American Economic Review*, 75 (3), June 1985, p. 424-440.

Lamberson, P.J., and Scott E. Page. "First Mover or Higher Quality? Optimal Product Strategy in Markets with Positive Feedbacks," *Journal of Economics and Management Strategy*, 27 (1), Spring 2018, p. 40-52.

Liebowitz, S. J., and Stephen E. Margolis. "Network Externality: An Uncommon Tragedy," *Journal of Economic Perspectives*, 8 (2), Spring 1994, p. 133-150.

Milgrom, Paul R., and Steven Tadelis. "How Artificial Intelligence and Machine Learning Can Impact Market Design," in Ajay Agrawal, Joshua Gans and Avi Goldfarb (eds), *The Economics of Artificial Intelligence: An Agenda*, Chicago: University of Chicago Press, 2019, p. 567-585.

Moulin, Herve. *The Strategy of Social Choice*, Amsterdam: North-Holland, 1983.

Obashi, Hiroshi. "The Role of Network Externalities in the U.S. VCR Market, 1978-1986," *Journal of Economics and Management Strategy*, 12 (4), Winter 2003, p. 447-494.

Rasmusen, Eric. *Games and Information*, 4th Edition, Malden, MA: Blackwell Publishing, 2007.

Rasmusen, Eric, Mark Ramseyer, and John Wiley Jr. "Naked Exclusion," *American Economic Review*, 81, December 1991, p. 1137-1145.

Segal, Ilya R. "Coordination and Discrimination in Contracting with Externalities: Divide and Conquer?", *Journal of Economic Theory*, 113 (2), December 2003, p. 147-181.

Segal, Ilya R., and Michael D. Whinston. "Naked Exclusion: Comment," *American Economic Review*, 90, March 2000, p. 296-309.

Shapiro, Carl, and Hal Varian. *Information Rules: A Strategic Guide to the Network Economy*, Boston, MA: Harvard Business School Press, 1999.

Shiller, Ben. "Personalized Price Discrimination Using Big Data," Working Paper 108, Brandeis University, Department of Economics and International Business School, 2016.

Weyl, Glen E. "A Price Theory of Multi-Sided Platforms," *American Economic Review*, 100 (4), September 2010, p. 1642-1672.

White, Alexander, and Glen E. Weyl. "Insulated Platform Competition," Working Paper, Tsinghua University, School of Economics and Management, April 2016.

Whinston, Michael D. "Exclusivity and Tying in U.S. v. Microsoft: What We Know and Don't Know," *Journal of Economic Perspectives*, 15, Spring 2001, p. 63-80.

APPENDIX

Proof of Proposition 2

(A) No subgame-perfect equilibrium that meets the monotonicity criterion in which supplier 2 sells to a strictly positive measure $x_2 \in (0,1]$ of customers exists.

Suppose to the contrary that there exists such an equilibrium with a strictly positive measure, $x_2 \in (0,1]$, of customers for supplier 2 and $\int_0^{x_2} p_2(j)dj \geq 0$ (so that supplier 2's pricing strategy is viable in that it leads to a non-negative profit). Then, according to the monotonicity criterion, it is rationally expected that supplier 1 will continue to sell to the measure $1-x_2$ of its equilibrium customers if it keeps prices $p_1(j)$, $\forall j \in [0,1-x_2]$ constant (and only makes possible changes in prices $p_1(j)$ for customers in x_2). Furthermore, given the pricing strategy of supplier 2 and given the expectation that a measure $1-x_2$ and x_2 of customers buy from suppliers 1 and 2, respectively, supplier 1 could poach a customer j from supplier 2 in stage 2 by offering a stage-1 price $p_1(j)$ such that $v_1(1-x_2)-p_1(j) \geq v_2(x_2)-p_2(j)$, i.e., $p_1(j) = p_2(j)+(\alpha_1-\alpha_2)+[n_1(1-x_2)-n_2x_2]$. Similarly, given that supplier 1 is expected to poach a measure $\delta \leq x_2$ of customers from supplier 2, supplier 1 can simultaneously poach a customer j in stage 2 by offering a stage-1 price $p_1(j) = p_2(j)+v_1(1-x_2+\delta)-v_2(x_2-\delta) = p_2(j)+(\alpha_1-\alpha_2)+\left[n_1(1-x_2+\delta)-n_2(x_2-\delta)\right]$.

It follows that given the pricing strategy of supplier 2, supplier 1 would be able to poach all the customers of supplier 2 in stage 2 by offering each customer $j \in [0,x_2]$ a price $p_1(j) = p_2(j)+v_1(1-x_2+j)-v_2(x_2-j)$ in stage 1. Specifically, given such stage-1 price offers from suppliers 1 and 2, each customer $j \in [0,x_2]$ would buy from supplier 1 since customers $j' < j$ would simultaneously (and always) also buy from supplier 1 and regardless of the buying decisions of customers $j'' > j$; supplier 1 exactly compensates a customer $j \in [0,x_2]$ for the difference in the reservation value of the two brands on the basis that customers $j' < j$ always buy from supplier 1. Thus all the customers of supplier 2 buy from supplier 1.

If supplier 1 followed the above pricing strategy, it would earn a profit from poaching the measure, x_2, of supplier 2's customers that would be equal to

$$\int_0^{x_2} [p_2(j)+v_1(1-x_2+j)-v_2(x_2-j)]dj =$$

$$\int_0^{x_2} p_2(j)dj + x_2[(\alpha_1-\alpha_2-n_2)+(1-\frac{1}{2}x_2)(n_1+n_2)]. \qquad \text{(A1)}$$

We can see that $(\alpha_1-\alpha_2-n_2)+(1-\frac{1}{2}x_2)(n_1+n_2)\geq(\alpha_1-\alpha_2-n_2)+\frac{1}{2}(n_1+n_2)=\frac{1}{2}(\Delta_1-\Delta_2)>0$ (given that $x_2\in(0,1]$). Then, since $\int_0^{x_2} p_2(j)dj\geq 0$, condition (A1) is strictly positive. It follows that there exists no subgame-perfect equilibrium that meets the monotonicity criterion in which in which a measure $x_2\in(0,1]$ of customers buy from supplier 2 and $\int_0^{x_2} p_2(j)dj\geq 0$ since given the pricing strategy of supplier 2 in such a hypothetical equilibrium, supplier 1 could follow at least one profitable pricing strategy, i.e., the above pricing strategy, to poach the equilibrium customers of supplier 2.

We can now see that the poaching strategy of supplier 1 carries through to possible corner solutions. In particular, the base poaching strategy of the previous paragraph is applicable if $\alpha_1+n_1(1-x_2+j)\geq p_2(j)-\Delta_2+(n_1+n_2)(1-x_2+j)\Rightarrow p_2(j)\leq\alpha_2+n_2(x_2-j)$, $\forall j\in[0,x_2]$. Then, supplier 1 offers each customer $j\in[0,x_2]$ a price $p_2(j)-\Delta_2+(n_1+n_2)(1-x_2+j)\leq\alpha_1+n_1(1-x_2+j)$. Such pricing by supplier 1 is feasible, i.e., it does not exceed the reservation value, $\alpha_1+n_1(1-x_2+j)$, of supplier 1's brand to customer j given that customers $j'<j$ buy from supplier 1 (the reservation value is effectively a ceiling on any price that supplier 1 may charge customer j).

For simplicity, suppose now that supplier 2 offers prices to customers $j\in[0,x_2]$ in weakly descending order (i.e., $p_2(j')\geq p_2(j'')$ if $j'<j''$).[22] Furthermore, suppose that $\exists j\in[0,x_2]$, for which $p_2(j)>\alpha_2+n_2(x_2-j)$; the price that supplier 1 would charge customer j in the base poaching strategy would not be feasible since it would strictly exceed the reservation value $\alpha_1+n_1(1-x_2+j)$. However, even in such a corner case, supplier 1 is able to profitably poach a strictly positive measure of customers from supplier 2. If $\underline{j}$ is the first $j\in[0,x_2]$ for which $p_2(j)>\alpha_2+n_2(x_2-j)$, supplier 1 is able to earn a strictly positive profit by poaching customers $j\in[0,\underline{j}]$ (i.e., the most lucrative customers that have been offered the highest prices) from supplier 2. In particular, a customer $j\in[0,\underline{j}]$ can be lured away if it is offered a price $\alpha_2+n_2(x_2-\underline{j})-\left[\Delta_2-(n_1+n_2)(1-x_2+j)\right]$ by supplier 1 (since $\alpha_2+n_2(x_2-\underline{j})<p_2(j)$). We can see that $[\alpha_2+n_2(x_2-\underline{j})]\underline{j}-\int_0^{\underline{j}}\left[\Delta_2-(n_1+n_2)(1-x_2+j)\right]dj=[\alpha_1+n_1(1-x_2)+0.5(n_1-n_2)\underline{j}]\underline{j}$ and $\alpha_1+n_1(1-x_2)+0.5(n_1-n_2)\underline{j}=0.5\underline{j}(\Delta_1-\Delta_2)+\alpha_2\underline{j}+\alpha_1(1-\underline{j})+n_1(1-x_2)>0$. Supplier 1

[22] Descending prices in $j\in[0,x_2]$ simplify the notation in regard to supplier 1's most lucrative poaching strategy. If supplier 1 tries to poach a subset $[0,j']$, $j'\in[0,x_2]$ of suppler 2's customers, it may optimally target the customers with the highest prices that can be more easily tempted to buy from supplier 1.

can even extract strictly higher prices from some customers $j \in [0, \underline{j}]$ since $p_2(j) > \alpha_2 + n_2(x_2 - j)$. Thus poaching all customers $j \in [0, \underline{j}]$ entails a strictly positive profit for supplier 1 since $\int_0^{\underline{j}} p_2(j) - \Delta_2 + (n_1 + n_2)j]dj > 0$. In any case, in this corner solution an equilibrium in which a measure x_2 of customers buy from supplier 2 does not exist since in such a hypothetical equilibrium supplier 1 can profitably poach at least $j \in [0, \underline{j}]$ customers from supplier 2.

Furthermore, the above analysis is exactly applicable to the corner case in which $\exists j \in [0, x_2]$, $p_2(j) > \alpha_2 + n_2(x_2 - j)$ (so that supplier 1 may charge customer j its full reservation value $\alpha_1 + n_1(1 - x_2 + j)$, rather than the interior poaching price $p_2(j) - \Delta_2 + (n_1 + n_2)(1 - x_2 + j)$, since poaching leads to a price above the reservation value for poached supplier 2). We saw above that $\alpha_1 + n_1(1 - x_2 + j) = p_2(j) - \Delta_2 + (n_1 + n_2)(1 - x_2 + j) \Rightarrow p_2(j) = \alpha_2 + n_2(x_2 - j)$. Thus the corner case in which $p_2(j) > \alpha_2 + n_2(x_2 - j)$ exactly corresponds to the already examined corner case in which $p_2(j) - \Delta_2 + (n_1 + n_2)(1 - x_2 + j) > \alpha_1 + n_1(1 - x_2 + j)$.

(B) A subgame-perfect equilibrium that meets the monotonicity criterion in which $x_1 = 1$ exists.

Suppose that there exists an equilibrium where supplier 1 sells to all customers, and $\int_0^1 p_1(j)dj > \int_0^1 [v_1(j) - v_2(1 - j)]dj = (\alpha_1 - \alpha_2) + \frac{(n_1 - n_2)}{2} = \frac{\Delta_1 - \Delta_2}{2} > 0$. Then, supplier 2 could profitably poach all the customers of supplier 1 by adopting a poaching strategy similar to supplier 1 in part A. In particular, in stage 1 supplier 2 could offer each customer $j \in [0,1]$ a price just a shade under $p_1(j) + v_2(j) - v_1(1 - j)$ (given the tie-breaking convention that if a customer is indifferent between the two suppliers, it buys from supplier 1), earning a profit just a shade under $\int_0^1 p_1(j)dj - \frac{\Delta_1 - \Delta_2}{2} > 0$). Thus such an equilibrium does not exist.

Furthermore, such an equilibrium (in which supplier 1 sells to all customers, and $\int_0^1 p_1(j)dj > \frac{\Delta_1 - \Delta_2}{2} > 0$) does not exist in case there are corner solutions. For simplicity, similar to part A suppose that supplier 1 offers prices to customers $j \in [0,1]$ in weakly descending order (i.e., $p_1(j') \geq p_1(j'')$ if $j' < j''$). Furthermore, suppose that $\exists j \in [0,1]$, for which $\alpha_2 + n_2 j < p_1(j) - \Delta_1 + (n_1 + n_2)j \Rightarrow p_1(j) > \alpha_1 + n_1(1 - j)$; the price that supplier 2 would charge customer j in the base poaching strategy would be infeasible since it would strictly exceed the reservation value $\alpha_2 + n_2 j$. However, even in such a corner case, supplier 2 is able to profitably poach a strictly positive measure of customers from supplier 2. If $\underline{j}$ is the first $j \in [0,1]$ for which $p_1(j) > \alpha_1 + n_1(1 - j)$, supplier 2 is able to earn a strictly positive profit by poaching customers $j \in [0, \underline{j}]$ (i.e., the most lucrative customers that have been offered the highest prices) from supplier 1. In particular, a customer $j \in [0, \underline{j}]$ can be lured away if it is offered a price $\alpha_1 + n_1(1-\underline{j}) - \left[\Delta_1 - (n_1 + n_2) j\right]$ by supplier 2 (since $\alpha_1 + n_1(1-j) < p_1(j)$). We can see that $[\alpha_1 + n_1(1-\underline{j})]\underline{j} - \int_0^{\underline{j}} [\Delta_1 - (n_1+n_2) j] dj = [\alpha_2 + 0.5(n_2 - n_1)]\underline{j} \geq 0$ (since in the base model we assume that $2\alpha_2 \geq n_1 - n_2$). Supplier 2 can even extract strictly higher prices from some customers $j \in [0, \underline{j}]$ since $p_1(j) > \alpha_1 + n_1(1-j)$. Thus poaching all customers $j \in [0, \underline{j}]$ entails a strictly positive profit for supplier 2 since $\int_0^{\underline{j}} p_1(j) - \Delta_1 + (n_1+n_2) j] dj > [\alpha_2 + 0.5(n_2 - n_1)]\underline{j} \geq 0$. In any case, in this corner solution an equilibrium in which all customers buy from supplier 1 does not exist since in such a hypothetical equilibrium supplier 2 can profitably poach at least $j \in [0, \underline{j}]$ customers.

Furthermore, the above analysis is exactly applicable to the corner case in which $\exists j \in [0,1]$, $p_1(j) > \alpha_1 + n_1(1-j)$ (so that supplier 2 may charge customer j its full reservation value $\alpha_2 + n_2 j$, rather than the interior poaching price $p_1(j) - \Delta_1 + (n_1 + n_2) j$, since poaching leads to a price above the reservation value for poached supplier 1). We saw above that $\alpha_2 + n_2 j = p_1(j) - \Delta_1 + (n_1 + n_2) j \Rightarrow p_1(j) = \alpha_1 + n_1(1-j)$. Thus the corner case in which $p_1(j) > \alpha_1 + n_1(1-j)$ exactly corresponds to the already examined corner case in which $p_1(j) - \Delta_1 + (n_1 + n_2) j > \alpha_2 + n_2 j$. It follows that in this corner solution an equilibrium in all customers buy from supplier 1 does not exist since supplier 2 can profitably poach at least $j \in [0, \underline{j}]$ customers from supplier 1 (similar to above).

Suppose now that in equilibrium supplier 1 sells to all customers and $\int_0^1 p_1(j) dj \leq \frac{\Delta_1 - \Delta_2}{2}$. Then, supplier 2 is unable to poach the entire range of customers $j \in [0,1]$ by following the benchmark poaching strategy since such a strategy would lead to a strictly negative profit for supplier 2 (i.e., to a profit strictly smaller than $\int_0^1 p_1(j) dj - \frac{\Delta_1 - \Delta_2}{2} \leq 0$). In addition, for such an equilibrium to exist supplier 1's prices must not leave room for the profitable poaching of any subset $[0, z]$ ($z \in [0,1]$) of customers (rather than all customers) by supplier 2. Thus in such an equilibrium supplier 1's prices meet the condition $\int_0^z p_1(j) dj + \int_0^z [v_2(j) - v_1(1-j)] dj \leq 0 \Rightarrow \int_0^z p_1(j) dj \leq z[\Delta_1 - 0.5z(n_1 + n_2)]$, $\forall z \in [0,1]$.[23]

[23] This condition also precludes (among other things) the occurrence of corner cases in which $\exists j \in [0,1]$, $p_1(j) > \alpha_1 + n_1(1-j)$. As we explain above, the occurrence of such corner cases would allow supplier 2 to poach a strictly positive measure of customers from supplier 1.

Several prices by supplier 1 meet such a condition (see, for example, note 14). It follows that there exist equilibria in which supplier 1 sells to all customers, and supplier 2 is unable to use the benchmark poaching strategy to profitably poach any customers.

In a different vein, we can see that in equilibria in which supplier 1 sells to all customers and $\int_0^z p_1(j)^* dj \le z[\Delta_1 - 0.5z(n_1+n_2)]$, $\forall z \in [0,1]$, supplier 2 is unable to follow any (and not just the benchmark) strategy to profitably poach supplier 1's customers. Suppose that supplier 2 changes the base poaching strategy by offering a customer j_A a price $p_1(j_A) - \Delta_1 + (n_1+n_2)j_A + \omega_A$, where $\omega_A > 0$ (i.e., supplier 2 applies to customer j_A a price increase ω_A compared with the base poaching strategy).[24] Thus in the order of supplier 2's stealing offers such a customer j_A is effectively shifted to $j_A + \omega_A/(n_1+n_2)$, leaving a void in $[0, j_A + \omega_A/(n_1+n_2))$. Then, $\forall j \in (j_A, j_A + \omega_A/(n_1+n_2))$, compared with an outcome in which a set of customers $S = \{[0, j_A) \cup (j_A, j]\}$ bought from supplier 2 (and the rest from supplier 1), customer j would be better off if it bought from supplier 1 as long as all customers $j \ge j$ bought from supplier 1 (since $S \subset [0, j]$).

It follows that there exists a stage-2 subgame in which given supplier 2's stage-1 strategy, all customers $j > j_A$ still buy from supplier 1; buying from supplier 1 is the optimal strategy for each customer $j > j_A$ given that all such customers buy from supplier 1. In this subgame supplier 2 is also unable to collect the higher price $p_1(j_A) - \Delta_1 + (n_1+n_2)j_A + \omega_A$ since customers at $j = j_A + \omega_A/(n_1+n_2)$ (where customer j_A has been effectively shifted) buy from supplier 1.[25] In such a stage-2 subgame supplier 2's pricing strategy leads to a strictly negative profit for supplier 2 since as proposition 1 states, supplier 1's prices meet the condition $\int_0^z p_1(j)^* dj \le z[\Delta_1 - 0.5z(n_1+n_2)]$, $\forall z \in [0,1]$ (and thus $\int_0^{j_A} p_1(j) dj \le j_A[\Delta_1 - 0.5 j_A(n_1+n_2)]$). It follows that there exists at least one stage-1 subgame in which supplier 2 refrains from adopting the poaching strategy of this paragraph (expecting that it will lead to negative profits in stage 2).

Suppose now that supplier 2 changes the benchmark poaching strategy by offering a customer j_A a price $p_1(j_A) - \Delta_1 + (n_1+n_2)j_A + \omega_A$, where $\omega_A > 0$ and a measure ϕ_B of customers a price $p_1(j_B) - \Delta_1 + (n_1+n_2)j_B - \omega(j_B)$, where $j_B \in [0, \phi_B]$,

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[24] If $\omega_A \le 0$, the poaching strategy would be weakly less profitable for supplier 2 than the benchmark poaching strategy (which is already unprofitable) if customer j_A bought from supplier 2.

[25] If $j_A + \omega_A/(n_1+n_2) > 1$, customer j_A is effectively shifted to $j = 1$. The rest of the analysis still applies. Furthermore, if $j_A = 1$, customer j_A buys from supplier 1, and thus supplier 2 is still unable to collect the higher price $p_1(j_A) - \Delta_1 + (n_1+n_2)j_A + \omega_A$.

$\omega(j_B) \geq 0$, and $\omega_A > \int_0^{\phi_B} \omega(j_B) dj_B$. Thus in the order of supplier 2's stealing offers a customer j_B is effectively shifted to $j_B - \omega(j_B)/(n_1+n_2)$, which can cover a possible void (generated, for example, by ω_A) in $[j_B - \omega(j_B)/(n_1+n_2), j_B)$. However, since $\omega_A > \int_0^{\phi_B} \omega(j_B) dj_B$, supplier 2's price reductions $\omega(j_B)$ cannot compensate for its price increase ω_A; there exists at least a subset (j', j'') of $(j_A, j_A + \omega_A/(n_1+n_2))$ (where $j' \geq j_A$, $j'' \leq j_A + \omega_A/(n_1+n_2))$ to which the previous analysis applies. In particular, $\forall j \in (j', j'')$, compared with an outcome in which a set of customers $S = \{[0, j') \cup (j', j]\}$ bought from supplier 2 (and the rest from supplier 1), customer j would be better off if it bought from supplier 1 as long as all customers $j \geq j$ bought from supplier 1. The same reasoning applies when supplier 2 changes the benchmark poaching strategy by offering a measure ϕ_A of customers a price $p_1(j_A) - \Delta_1 + (n_1+n_2) j_A + \omega(j_A)$, where $j_A \in [0, \phi_A]$, $\omega(j_A) \geq 0$, and a measure ϕ_B of customers a price $p_1(j_B) - \Delta_1 + (n_1+n_2) j_B - \omega(j_B)$, where $j_B \in [0, \phi_B]$, $\omega(j_B) \geq 0$, and $\int_0^{\phi_A} \omega(j_A) dj_A > \int_0^{\phi_B} \omega(j_B) dj_B$. It follows that there exists a stage-2 subgame in which supplier 2's pricing strategy leads to a strictly negative profit for supplier 2 and thus a stage-1 subgame in which supplier 2 refrains from adopting such a poaching strategy.

(C) Possible subgame-perfect equilibria that meet the monotonicity criterion.

It follows from parts A and B of the proof that in a subgame-perfect equilibrium that meets the monotonicity criterion supplier 2 never sells to a strictly positive measure of customers. There exist, on the other hand, stage-1 subgames in which supplier 2 does not attempt to poach supplier 1's customers. Thus in a subgame-perfect equilibrium that meets the monotonicity criterion supplier 1 always sells to all customers, i.e., $x_1^* = 1$ and $x_2^* = 0$. There are multiple possible equilibrium prices so that $0 \leq \int_0^z p_1(j)^* dj \leq \int_0^z [v_1(1-j) - v_2(j)] dj = z[\Delta_1 - \frac{z(n_1+n_2)}{2}]$, $\forall z \in [0,1]$.

<u>Proof of Proposition 3</u>

The equilibrium outcome is socially optimal when supplier 1, which always sells to all customers (proposition 2), has a brand with a larger reservation value than supplier 2 in a winner-takes-all outcome, i.e., when $v_1(1) - v_2(1) = \alpha_1 + n_1 - \alpha_2 - n_2 \geq 0$. Thus the social optimum fails to materialize when both $\Delta_1 > \Delta_2$ (which holds by assumption) and $v_1(1) < v_2(1)$, i.e., when $n_2 > n_1$ and $\frac{n_2 - n_1}{2} < \alpha_1 - \alpha_2 < n_2 - n_1$. The equilibrium outcome is socially optimal for all parameters outside this range.

Proof of Proposition 4

If supplier i ($i \in \{1,2\}$) has sold to a measure $\overline{x_i}$ of customers, the reservation value of supplier i's brand is weakly larger even if all the remaining measure $1-\overline{x_i}$ of customers buy from supplier $-i \neq i$. In particular,

$$\alpha_1 + n_1\overline{x_1} = \alpha_2 + n_2(1-\overline{x_1}) \Rightarrow \overline{x_1} = \frac{\Delta_2}{n_1+n_2}, \tag{A2a}$$

$$\alpha_2 + n_2\overline{x_2} = \alpha_1 + n_1(1-\overline{x_2}) \Rightarrow \overline{x_2} = \frac{\Delta_1}{n_1+n_2}. \tag{A2b}$$

Since $\Delta_1 > \Delta_2$, conditions (A2a) and (A2b) imply that $\overline{x_1} < 0.5 < \overline{x_2} = 1-\overline{x_1}$.

Suppose that in a subgame suppliers 1 and 2 have sold to a measure x_1 and x_2 of customers, respectively, where $\overline{x_1} - x_1 < \overline{x_2} - x_2$ (or $\overline{x_1} - \overline{x_2} < x_1 - x_2$). Then, if $x_1 = \overline{x_1}$, supplier 1 will sell to entire measure $1-\overline{x_1}-x_2$ of remaining customers. Even if all customers $j \in (\overline{x_1}+x_2, 1)$ bought from supplier 2, condition (A2a) implies that customer $j=1$ would always buy from supplier 1. If, for example, supplier 1 offered a price equal to zero, supplier 2 would be unable to profitably counter supplier 1's price offer. Thus according to backward induction, given that each customer $j \in (\overline{x_1}+x_2, 1)$ expects all subsequent customers $j" > j$ to buy from supplier 1, it buys from supplier 1. Supplier 1 charges a price $p_1(j) = \Delta_1 - x_2(n_1+n_2)$, $\forall j \in (\overline{x_1}+x_2, 1]$ (since we have assumed a zero price squeeze, $\rho = 0$, by the losing supplier).

Suppose now that $x_1 = \overline{x_1} - \varepsilon$, where ε is infinitesimal, i.e., $\varepsilon \to 0$.[26] If supplier 1 sells to a measure ε of customers, it will then sell to all subsequent customers (as the previous paragraph implies) earning a subsequent profit $\Pi_1(x_1+\varepsilon, x_2) = [\Delta_1 - x_2(n_1+n_2)](1-x_1-\varepsilon-x_2)$, or $\Pi_1(x_1, x_2) = [\Delta_1 - x_2(n_1+n_2)](1-x_1-x_2)$ since $\varepsilon \to 0$. Suppose, however, that competing supplier 2 also expects that if it sells to ε, it will be able to sell to all subsequent customers. Even in this case, the maximum subsequent profit that supplier 2 can possibly earn is $\Pi_2(x_1, x_2) = [\Delta_2 - x_1(n_1+n_2)](1-x_1-x_2)$ (when each customer $j \in (x_1+x_2+\varepsilon, 1]$ expects that all customers $j \in (x_1+x_2+\varepsilon, 1]$ will buy from supplier 2). We can see that

$$\Pi_1(x_1, x_2) - \Pi_2(x_1, x_2) = [\Delta_1 - \Delta_2 + (x_1-x_2)(n_1+n_2)](1-x_1-x_2) > 0. \tag{A3}$$

Condition (A3) is strictly positive because $\overline{x_1} - \overline{x_2} < x_1 - x_2$ (and $\Pi_1(\overline{x_1}, \overline{x_2}) - \Pi_2(\overline{x_1}, \overline{x_2}) = 0$). It follows that there exist no equilibrium in which supplier 2

[26] This is the equivalent of having sold to exactly one customer less than the threshold $\overline{x_1}$ in a game with a finite number of customers. A continuum is a limit case as the number of customers approaches infinity.

sells to ε because it can always be profitably countered by supplier 1 (with at least one profitable counter). For example, supplier 1 is able to offer ε a price as low as $-\Pi_1(x_1,x_2)/\varepsilon$, to which supplier 2 is unable to respond successfully since $(\Pi_1(x_1,x_2)-\Pi_2(x_1,x_2))/\varepsilon+\alpha_1+n_1(1-x_2)-\alpha_2-n_2(1-x_1)>0$.[27]

According to backward induction, such reasoning implies that all customers $j\in[0,1]$ buy from supplier 1. In particular, for customer $j=0$ we have $x_1=x_2=0$ and thus $\overline{x_1}-x_1<\overline{x_2}-x_2$. Then, the above argument carries through. Agents expect that if supplier 1 sells to $j=0$, it will also sell to all subsequent customers. There exist no equilibrium in which supplier 2 sells to $j=0$ (or any other j) since for $\varepsilon\to 0$ we have $(\Pi_1(x_1,x_2)-\Pi_2(x_1,x_2))/\varepsilon+\alpha_1+n_1-\alpha_2-n_2>0$. It follows that in a subgame-perfect equilibrium all customers $j\in[0,1]$ always buy from supplier 1 at a price $p_1(j)=\Delta_1$, $\forall j\in[0,1]$ (while $p_2(j)=0$). Our results also carry through to any degree of price squeeze $\rho\in[0,1]$ (rather than only to $\rho=0$) by the losing supplier. Then, in a subgame-perfect equilibrium all customers $j\in[0,1]$ always buy from supplier 1 at a price $p_1(j)=(1-\rho)\Delta_1$, $\forall j\in[0,1]$. Furthermore, our results carry through even if the two suppliers do not face the same degree of price squeeze in case they dominate the market as long as supplier 2 does not face a substantially smaller price squeeze than supplier 1.[28]

Some Additional Extensions of the Model

(i) Differences in Unit Costs

As is common in the literature (e.g., Katz and Shapiro (1985), Jullien (2011), Halaburda, Jullien and Yehezkel (2020)), in the base model suppliers 1 and 2 have the same unit cost $c=0$. Such an assumption allows us to present our argument clearly and is also relevant to several high-technology network industries since such industries tend to have negligible marginal costs once a product has been developed and introduced into the market (e.g., Shapiro and Varian (1999)). Our analysis directly carries through when supplier i ($i\in\{1,2\}$) has a unit cost c_i since it is straightforward to extend condition (2) to incorporate costs. Since the dominant value margin, Δ_i, of supplier i is the difference between the social value of a sale to a customer by the fully dominant brand i and the fully dominated brand $-i$, condition (2) becomes $\Delta_i=v_i(1)-v_{-i}(0)-c_i+c_{-i}=\alpha_i+n_i-\alpha_{-i}-c_i+c_{-i}$, where $i\in\{1,2\}$ and $-i\neq i$. In the spirit of the base model (where $\alpha_i>0$, $i\in\{1,2\}$) it is assumed that a supplier's net stand-alone effect — i.e., the difference between the stand-alone benefit and the unit cost, — is

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[27] Since $\varepsilon\to 0$, subsequent supplier profits, $\Pi_1(x_1,x_2)$ or $\Pi_2(x_1,x_2)$, are generated from a vastly superior measure of customers compared to ε. Thus the difference, $(\Pi_1(x_1,x_2)-\Pi_2(x_1,x_2))/\varepsilon$, in subsequent supplier profits that can be possibly channeled into capturing ε dwarfs the difference, $\alpha_1+n_1(1-x_2)-\alpha_2-n_2(1-x_1)$, in ε's own expected reservation values.

[28] If supplier 2 is expected to face a considerably smaller degree of price squeeze than supplier 1 in case it dominates, it may be able to counter supplier 1's early price offers and actually prevail despite its lower dominant value margin. The reason is that supplier 2 is expected to gain more from its dominance.

strictly positive, i.e., $\alpha_i - c_i > 0$, $i \in \{1,2\}$. Furthermore, as in the base model, both suppliers are viable ($\Delta_i > 0$, $i \in \{1,2\}$), and $\Delta_1 > \Delta_2$.

By following the same procedure as in the base model, we can see that supplier 1 with the larger dominant value margin ($\Delta_1 - \Delta_2 = 2(\alpha_1 - \alpha_2) + n_1 - n_2 - 2(c_1 - c_2) > 0$) always sells to all customers in a subgame-equilibrium that meets the monotonicity criterion. Thus unit costs disproportionately (relative to network benefits) impact a supplier's ability to implement profitable divide-and-conquer poaching strategies in the same way that stand-alone benefits do (given that they are both parts of a supplier's net stand-alone effect). A lower unit cost (similar to a larger stand-alone benefit) constitutes a disproportionately important advantage in the struggle for prevalence.

(ii) Non-linear Network Benefits

In the base model the network benefits, $n_i x_i$, of each supplier $i \in \{1,2\}$, are linear, which allows us to bring out our argument in a clear and straightforward manner. Suppose now that the functional forms of the network benefits of supplier i are more general, i.e., $n_i(x)$, where $n_i'(x) > 0$ and $n_i(0) = 0$. Then, our result that the supplier 1 with the larger dominant value margin ($2(\alpha_1 - \alpha_2) + [n_1(1) - n_2(1)] > 0$) always sells to all customers in equilibrium carries through as long as the functional forms of network benefits are sufficiently well-behaved, i.e., $n_1(x)$ is not much more convex than $n_2(x)$, or $n_1''(x) - n_2''(x)$ is not too much larger than zero (at least in a large range of $x \in [0,1]$).

<< INSERT FIGURE 2 HERE >>

As in the base model, the presence of network externalities, as well as the use of the monotonicity criterion as an equilibrium refinement, leads to a winner-takes-all outcome. Furthermore, by following the same procedure as in the base model, we can see that supplier 1 always sells to all customers $j \in [0,1]$ as long as $\alpha_1 - \alpha_2 + \int_0^1 [n_1(j) - n_2(1-j)]dj = \alpha_1 - \alpha_2 + \int_0^1 [n_1(j) - n_2(j)]dj \geq 0$. Such a condition holds as long as $n_1(x)$ is not much more convex than $n_2(x)$. Specifically, $\alpha_1 - \alpha_2 + \int_0^1 [n_1(j) - n_2(j)]dj$ corresponds graphically to the area under $p_1(j) - p_2(j) = -\alpha_2 - n_2(1-j) + \alpha_1 + n_1(j)$ in supplier 1's counter in case supplier 2 sells to all customers. In figure 2 we can see that $p_1(j) - p_2(j)$ has a positive upper bound, $\Delta_1 = \alpha_1 + n_1(1) - \alpha_2$, on the right ($j = 1$) which has a larger absolute value than the negative lower bound, $-\Delta_2 = -(\alpha_2 + n_2(1) - \alpha_1)$, on the left ($j = 0$). The area under $p_1(j) - p_2(j)$ is weakly positive as long as $n_1(j) - n_2(j)$ is not too convex.[29] Figure 2 shows an example in which $n_1(j) - n_2(j)$ is too convex and thus our results do not carry

[29] For a given $n_1(0) = 0$ and $n_1(1) > 0$, the area under $n_1(j)$, $j \in [0,1]$, i.e., $\int_0^1 n_1(j)dj$, is smaller as $n_1(j)$ becomes more convex. The same for $n_2(j)$.

through. However, even in this case the outcome of the game is deterministic. If network benefits $n_1(x)$ and $n_2(x)$ are not well-behaved, the results of the base model are exactly reversed; in all subgame-perfect equilibria that meet the monotonicity criterion all customers buy from supplier 2 (which has the smaller dominant value margin).

(iii) The Case in which $2\alpha_2 < n_1 - n_2$.

In the base model we assume that $2\alpha_2 \geq n_1 - n_2$, which allows us to obtain an interior solution regarding the upper bound, $\frac{\Delta_1 - \Delta_2}{2}$, of supplier 1's profit (proposition 2).[30] However, our results are even stronger if $2\alpha_2 < n_1 - n_2$ in the sense that the upper bound of supplier 1's profit is higher. In particular, as in the base model, there exists no subgame-perfect equilibrium that meets the monotonicity criterion in which supplier 2 sells to a strictly positive measure of customers. Part A of the proof of proposition 2 does not utilize the assumption that $2\alpha_2 \geq n_1 - n_2$ and is thus identical when $2\alpha_2 < n_1 - n_2$.

Furthermore, we can see that there are possible prices $p_1(j)$, $j \in [0,1]$ that allow supplier 1 to attain the upper bound, $\frac{\Delta_1 - \Delta_2}{2}$, of its base model profit, while supplier 2'a poaching strategy generates a strictly negative profit (rather than exactly a zero profit as in the base model). Suppose, for example, that $p_1(j) = \frac{\Delta_1 - \Delta_2}{2}$, $\forall j \in [0,1]$. Then, for $j \in [1 + \frac{\alpha_2 + 0.5(n_2 - n_1)}{n_1}, 1]$, we have $p_1(j) - \Delta_1 + (n_1 + n_2) j > \alpha_2 + n_2 j$. In this range the prices that would be offered by supplier 2 in the base poaching strategy are constrained by the reservation value of supplier 2's brand to customers; supplier 2 offers a customer $j \in [1 + \frac{\alpha_2 + 0.5(n_2 - n_1)}{n_1}, 1]$ a price $\alpha_2 + n_2 j < p_1(j) - \Delta_1 + (n_1 + n_2) j$.[31] Such a constraint makes the poaching strategy of supplier 2 less lucrative, leading to a strictly negative poaching profit $\int_0^1 p_1(j)dj - \Delta' < \int_0^1 p_1(j)dj - \frac{\Delta_1 - \Delta_2}{2} = 0$. Supplier 1 may thus be able to avoid poaching by supplier 2 even if $\int_0^1 p_1(j)dj > \frac{\Delta_1 - \Delta_2}{2}$; for example, supplier 1 may charge customers a uniform price that is higher (but not too much higher) than $\frac{\Delta_1 - \Delta_2}{2}$. There exists a $\overline{\Pi} \in (\frac{\Delta_1 - \Delta_2}{2}, \Delta_1)$ so that supplier 1's equilibrium profit is $0 \leq \Pi_1^* \leq \overline{\Pi}$.

[30] The sequential model in section 3.4, on the other hand, does not use the assumption that $2\alpha_2 \geq n_1 - n_2$.

[31] Furthermore, as part B of the proof of proposition 2 shows, when $2\alpha_2 < n_1 - n_2$, the occurrence of such corner cases for supplier 2's poaching prices does not imply that supplier 2 can profitably poach all customers $j \in [0, 1 + \frac{\alpha_2 + 0.5(n_2 - n_1)}{n_1}]$ (unlike the base model where $2\alpha_2 \geq n_1 - n_2$).

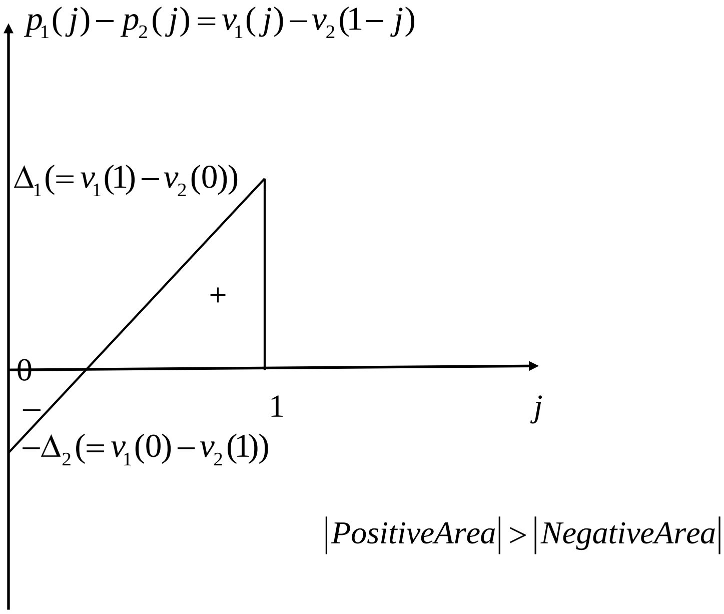

Figure 1: Price differences ($p_1(j) - p_2(j)$, $j \in [0,1]$) in supplier 1's possible counter.

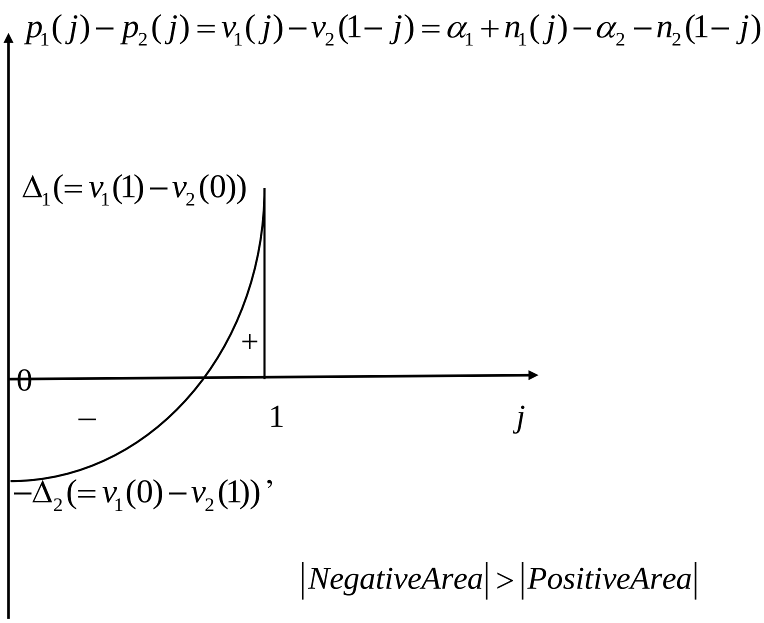

Figure 2: Example of supplier 1's unprofitable counter.